

# A New Attitude Control Method Reveals Relationship Between Stabilization Time and Input Constraints

Haichao Zhang, Xiaoqin Sun, Xueyi Liu, Bing Xiao<sup>+</sup>, and Kejun Dong

School of Automation, Northwestern Polytechnical University, Xi'an 710072, China

**Abstract.** This paper presents a new control method for satellite attitude control systems with input constraints. By augmenting a bounded hyperbolic tangent function into the control variables, the attitude control system is transformed to a higher-order configuration, thereby solving the input constraint issue. The obtained controller enables finite-time stability for the attitude control system. It demonstrates that the convergence time of the state trajectory is inversely proportional with the input constraints. The efficacy of the proposed approach is validated through simulation examples.

**Keywords:** Attitude control, Finite-time stability, Input constraints, Rigid satellite.

## 1. Introduction

In recent years, the widespread application of rigid satellites in space missions has drawn substantial attention to the issue of attitude control. Finite-time control schemes enable the controlled system to achieve stability within a finite time horizon. Compared with asymptotic control methods, finite-time control methods allow the closed-loop system to have stronger disturbance rejection capabilities and higher control precision. A variety of finite-time attitude control methods have thus been proposed [1], [2], [3]. However, none of these methods have considered the constraints on control torque. In practice, the available control torque is subject to amplitude constraints of actuators. Ignoring these constraints during the controller design may lead to degraded attitude performance or even instability of the closed-loop system.

To address the issue of input constraint of attitude control system, finite-time control schemes have been proposed. A saturated finite-time control law was reported by using homogeneous method, and can ensure the closed-loop system has finite-time stability [4]. To address the attitude synchronization problem of multiple spacecraft under the presence of angular velocity and input constraints in [5], an auxiliary system was proposed to tackle these constraints, and a finite-time command-filtered backstepping controller was designed to track the attitude of the leader spacecraft. By employing nonlinear transformations and bounded function models, the input constraint issue of the spacecraft attitude control system was converted into a state constraint problem. A finite-time attitude control scheme was then proposed based on the backstepping approach and robust control techniques in [6]. The saturation nonlinearity error was treated as a composite external disturbance, and a disturbance observer was employed to estimate it, thereby addressing the input saturation issue. Under the proposed controller, the closed-loop stability analysis indicates that the system state converges within a finite time [7]. Using the similar method to address the input constraints issue [8], an adaptive saturated pose tracking control law was designed, the estimated disturbances are directly used.

It is important to emphasize that for the existing input-constrained finite-time control schemes, the upper bound of the system's stabilization time is only related to the initial state of the system and control gains. However, if a system has a larger control authority, the converge time will also be smaller. For example, for a system  $\dot{x} = u$ , if the control input satisfies  $|u| \leq k = 2$ , the convergence time it takes for the system to converge from its initial value  $x_0 = 4$  to the origin is 2 seconds. The larger the  $k$ , the smaller convergence time. Therefore, a finite-time attitude control scheme for a rigid satellite with input constraints is proposed in this work. We approximate the actuator dynamics by using a bounded function and extend the attitude control system to a third-order one. For the augmented system, the adding a power integrator control

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<sup>+</sup> Corresponding author. Tel.: + 86-18697985540.  
E-mail address: xiaobing@nwpu.edu.cn.

technique is used to design an input-saturated attitude controller, thereby ensuring the finite-time stability of the controlled system. A highlight of this method is that it reveals, for the first time, the inverse proportional relationship between the system's stabilization time upper bound and the input constraint value.

## 2. Preliminaries

### 2.1. Satellite Attitude Control System

The attitude information of the satellite is described by Modified Rodrigues Parameters (MRPs). The attitude kinematics and dynamics equations of the satellite are described as [1]

$$\begin{cases} \dot{\boldsymbol{\sigma}} = \frac{1}{2} \left[ \left( \frac{1 - \boldsymbol{\sigma}^T \boldsymbol{\sigma}}{2} \right) \mathbf{I}_3 + \boldsymbol{\sigma}^\times + \boldsymbol{\sigma} \boldsymbol{\sigma}^T \right] \boldsymbol{\omega} = \mathbf{T}(\boldsymbol{\sigma}) \boldsymbol{\omega} \\ \mathbf{J} \dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \mathbf{J} \boldsymbol{\omega} + \boldsymbol{\tau} \end{cases} \quad (1)$$

where  $\boldsymbol{\sigma} \in \mathbb{R}^3$  and  $\boldsymbol{\omega} \in \mathbb{R}^3$  are attitude MRPs and the angular velocity of the satellite.  $\mathbf{I}_3$  is a  $3 \times 3$  dimensional identity matrix.  $\mathbf{J} \in \mathbb{R}^{3 \times 3}$  is the inertia matrix of the satellite.  $\boldsymbol{\sigma}^\times \in \mathbb{R}^{3 \times 3}$  and  $\boldsymbol{\omega}^\times \in \mathbb{R}^{3 \times 3}$  are antisymmetric matrices respect to the vectors  $\boldsymbol{\sigma}$  and  $\boldsymbol{\omega}$ .  $\boldsymbol{\tau} \in \mathbb{R}^3$  is the control torque. Considering that practical satellite systems are usually affected by control input constraint due to the physical limitations of on-board actuators, the control torque is bounded by  $|\tau_i| \leq \tau_{\max}$ ,  $i = 1, 2, 3$ , where  $\tau_{\max}$  is a known positive constant representing the maximum control amplitude of the actuators.

### 2.2. Lemmas and Assumption

Lemma 1 [9]: For a real-valued function  $\varphi(x, y) > 0$  and positive constants  $m$  and  $n$ , the next inequality holds:  $|x|^m |y|^n \leq \frac{m}{m+n} \varphi(x, y) |x|^{m+n} + \frac{n}{m+n} \varphi^n(x, y) |y|^{m+n}$ .

Lemma 2 [10]: For  $x \in \mathbb{R}$ ,  $y \in \mathbb{R}$ , and positive scalar  $p = a/b \leq 1$  with  $a$  and  $b$  being positive odd integers, it holds:  $|x^p - y^p| \leq 2^{1-p} |x - y|^p$ .

Lemma 3 [1]: For any  $0 < p \leq 1$  and  $\alpha_i \in \mathbb{R}$ , one has  $(|\alpha_1| + |\alpha_2| + \dots + |\alpha_i|)^p \leq |\alpha_1|^p + |\alpha_2|^p + \dots + |\alpha_i|^p$ .

Assumption 1 [10]: The angular velocity  $\boldsymbol{\omega} \in \mathbb{R}^3$  satisfies  $|\omega_i| \leq 1$  for any  $t > 0$ .

## 3. Finite-time Attitude Controller Design

A saturated attitude controllers will be developed to complete the desired maneuver within a finite time for the system (1). The finite-time stability of the controlled system is proved by using Lyapunov theory.

**Theorem :** For the attitude control system (1) with Assumption 1, a continuous, bounded function  $\boldsymbol{\tau} = \tau_{\max} \tanh(\mathbf{u})$  is adopted to approximate the actuator's dynamics. If the control signal  $\mathbf{u}$  is updated by

$$\dot{\mathbf{u}} = -(\mathbf{J}^{-1} \frac{\partial \tanh(\mathbf{u})}{\partial \mathbf{u}})^{-1} \boldsymbol{\Psi} \quad (2)$$

with  $\boldsymbol{\Psi} \in \mathbb{R}^3$ .  $\Psi_k = \zeta_k^{q_1} \psi_{3k}$  with  $\psi_{3k} = \beta_1 + \frac{m_2}{\tau_{\max}} + \frac{\varphi_2}{\tau_{\max}}$  and  $k = 1, 2, 3$ . Let  $\mu = \frac{t_1}{t_2} \in (0, \frac{1}{3})$ ;  $t_1$  is a positive even constant;  $t_2$  is a positive odd constant; two variables  $q_1 = 1 - 3\mu$  and  $\beta_1 > 0$  are defined; the definition of variables  $m_2$ ,  $\varphi_2$ , and  $\zeta_k$  will be given later; the following results will be obtained: (1) The control input signals are within the input constraints, i.e.,  $|\tau_i| < \tau_{\max}$ ; (2) The desired maneuver will be achieved within a finite time  $T_s$ , which is determined by  $T_s \leq \frac{1}{\tau_{\max}} \frac{1}{\mathcal{G}(1-\varrho)} V_3(0)^{1-\varrho}$  with  $\varrho$  and  $\mathcal{G}$  being the positive constants and  $V_3(0)$  being a initial value of Lyapunov function  $V_3$ . The states' convergence time is explicitly inversely proportional to the input constraints.

**Proof:** The proof procedure is divided into three steps within the framework of Lyapunov theory.

*Step 1:* Choose a Lyapunov function as  $V_1 = 0.5 \boldsymbol{\sigma}^T \boldsymbol{\sigma}$ . Remembering the property of  $\mathbf{T}(\boldsymbol{\sigma})$  [1], i.e.,  $\boldsymbol{\sigma}^T \mathbf{T}(\boldsymbol{\sigma}) \boldsymbol{\omega} = \Phi \boldsymbol{\sigma}^T \boldsymbol{\omega}$ , with  $\Phi = (1 + \boldsymbol{\sigma}^T \boldsymbol{\sigma}) / 4$  in mind and evaluating the time derivative of  $V_1$ , one has

$$\dot{V}_1 = \sigma^T \mathbf{T}(\sigma) \omega = \Phi \sigma^T \omega^* + \Phi \sigma^T (\omega - \omega^*) \quad (3)$$

The virtual law is designed as  $\omega_k^* = -\sigma_k^{q_2} \psi_{1k}$  with  $\psi_{1k} = 4(1 + \frac{3q_2}{\alpha} + h_1 \beta_1)$ , where  $h_1$ ,  $q_2$ , and  $\beta_1$  are positive scalar such that  $h_1 \geq \tau_{\max}$  and  $q_2 = 1 - \mu$ . The equation (3) is simplified as

$$\dot{V}_1 \leq -\sum_{k=1}^3 \sigma_k^\alpha \psi_{1k} + \sum_{k=1}^3 \Phi \sigma_k (\omega_k - \omega_k^*) \quad (4)$$

the last term of (4) can be simplified as  $\Phi \sigma_k (\omega_k - \omega_k^*) \leq \frac{1}{4} |\sigma_k|^\alpha + m_1 |\xi_k|^\alpha$  with  $\alpha = 2 - \mu$ ,  $\xi_k = \omega_k^{\frac{1}{q_2}} - \omega_k^{*\frac{1}{q_2}}$ , and  $m_1 = \Phi 2^{1-q_2} \frac{q_2}{\alpha} (2^{q_2-1} \frac{\alpha}{4\Phi})^{\frac{1}{q_2}}$ .  $\dot{V}_1$  is rewritten as  $\dot{V}_1 \leq -\sum_{k=1}^3 \sigma_k^\alpha (h_1 \beta_1 + \frac{3}{4} + \frac{3q_2}{\alpha}) + m_1 |\xi_k|^\alpha$  with  $\alpha = 2 - \mu$ .

*Step 2:* Select a Lyapunov candidate as  $V_2 = V_1 + \sum_{k=1}^3 Y_k$ .  $Y_k$  is given as  $Y_k = \int_{\omega_k^*}^{\omega_k} (\delta^{\frac{1}{q_2}} - \omega_k^{*\frac{1}{q_2}})^{2-q_2} d\delta$ .

Referring to [9],  $Y_k$  has the following properties  $\frac{\partial Y_k}{\partial \sigma_k} = -(2 - q_2) \frac{\partial (\omega_k^{\frac{1}{q_2}})}{\partial \sigma_k} \int_{\omega_k^*}^{\omega_k} (\delta^{\frac{1}{q_2}} - \omega_k^{*\frac{1}{q_2}})^{1-q_2} d\delta$  and

$\frac{\partial Y_k}{\partial \omega_k} = \xi_k^{2-q_2}$ . Using them, one has  $\dot{Y}_k = -(2 - q_2) \frac{d(\omega_k^{\frac{1}{q_2}})}{dt} \int_{\omega_k^*}^{\omega_k} (\delta^{\frac{1}{q_2}} - \omega_k^{*\frac{1}{q_2}})^{1-q_2} d\delta + \xi_k^{2-q_2} \dot{\omega}_k$ . Referring to [1],

the inequality holds  $\int_{\omega_k^*}^{\omega_k} (\delta^{\frac{1}{q_2}} - \omega_k^{*\frac{1}{q_2}})^{1-q_2} d\delta \leq |\xi_k|^{1-q_2} |\omega_k - \omega_k^*|$ . Differentiating  $\omega_k^{*\frac{1}{q_2}}$  with respect to time yields

$$-\frac{d(\omega_k^{*\frac{1}{q_2}})}{dt} = (\psi_{1k}^{\frac{1}{q_2}} + \frac{4}{q_2} \psi_{1k}^{\frac{1}{q_2}-1} c_1 \beta_0 \lambda \sigma_k^\lambda) [\mathbf{T}(\sigma) \omega]_k \leq \Phi (\psi_{1k}^{\frac{1}{q_2}} + \frac{4}{q_2} \psi_{1k}^{\frac{1}{q_2}-1} c_1 \beta_0 \lambda \sigma_k^\lambda) \sum_{i=1}^3 |\omega_i| \quad (5)$$

with the inequality  $[\mathbf{T}(\sigma) \omega]_k \leq \Phi (|\omega_1| + |\omega_2| + |\omega_3|)$  in [1] being used. Using Lemmas 1 and 2, for any  $i = 1, 2, 3$ , we have

$$\begin{aligned} & \gamma_k |\xi_k|^{1-q_2} |\omega_k - \omega_k^*| |\omega_i| \leq 2^{2-q_2} \gamma_k |\xi_k| |\xi_i|^{q_2} + 2^{1-q_2} \gamma_k |\xi_k| |\omega_i^*| \\ & \leq \gamma_k 2^{2-2q_2} \left( \frac{1}{\alpha} |\xi_k|^\alpha + \frac{q_2}{\alpha} |\xi_i|^\alpha \right) + \frac{1}{\alpha} (2^{3-q_2} \gamma_k)^\alpha \psi_{1i}^\alpha |\xi_k|^\alpha + \frac{q_2}{\alpha} |\sigma_i|^\alpha \end{aligned} \quad (6)$$

with  $\gamma_k = (2 - q_2) \Phi (\psi_{1k}^{\frac{1}{q_2}} + \frac{4}{q_2} \psi_{1k}^{\frac{1}{q_2}-1} c_1 \beta_0 \lambda \sigma_k^\lambda)$ . Invoking equation (5), the second term of  $\dot{V}_2$  is given as

$$\sum_{k=1}^3 \dot{Y}_k \leq \sum_{k=1}^3 n_k |\xi_k|^\alpha + \frac{3q_2}{\alpha} \sum_{k=1}^3 |\sigma_k|^\alpha + \sum_{k=1}^3 \xi_k^{2-q_2} \dot{\omega}_k \quad (7)$$

with  $n_k = \frac{1}{\alpha} (2^{3-q_2} \gamma_k)^\alpha \sum_{i=1}^3 \psi_{1i}^\alpha + \frac{3q_2}{\alpha} \gamma_k 2^{2-2q_2} + \frac{3}{\alpha} \gamma_k 2^{2-2q_2}$ . Due to the fact  $\|\omega^\times\| = \|\omega\|$ , assume  $\mathbf{H} = -\mathbf{J}^{-1} \omega^\times \mathbf{J} \omega$

such that  $\|\mathbf{H}\| \leq b_j \|\omega\|^2$  with  $b_j = \|\mathbf{J}\| \|\mathbf{J}^{-1}\|$  and  $\mathbf{H} \in \mathbb{R}^3$ . Therefore, one has  $H_k \leq b_j \sum_{i=1}^3 |\omega_i|^2$ .

In view of the attitude control system (1), one has  $\xi_k^{2-q_2} \dot{\omega}_k \leq \xi_k^{2-q_2} (|H_k| + \tau_k - \tau_k^*) + \xi_k^{2-q_2} \tau_k^*$ . Define a new variable  $\zeta_k = \tau_k^{\frac{1}{q_3}} - \tau_k^{*\frac{1}{q_3}}$ . Recalling Lemma 1 and Lemma 2, one has

$$\xi_k^{2-q_2} (\tau_k - \tau_k^*) \leq 2^{1-q_3} |\xi_k|^{2-2q_2} |\zeta_k|^{q_3} \leq \frac{1}{2} |\xi_k|^\alpha + m_2 |\zeta_k|^\alpha \quad (8)$$

with  $m_2 = 2^{1-q_3} \frac{q_3}{\alpha} (2^{q_3-1} \frac{\alpha}{2(2-q_2)})^{\frac{2-q_2}{q_3}}$ . Employing Assumption 1, one can obtain

$$|\xi_k|^{2-q_2} |H_k| \leq b_J |\xi_k|^{2-q_2} \sum_{i=1}^3 |\omega_i|^{q_2} \quad (9)$$

Using Lemmas 1 and 2, for any  $i = 1, 2, 3$ , one has

$$|\xi_k|^{2-q_2} |\omega_i|^{q_2} \leq \phi_1 |\xi_k|^\alpha + 2^{1-q_3} \frac{q_3}{\alpha} |\xi_i|^\alpha + \eta_1 \sigma_k^{q_3} |\xi_k|^{2-q_2} \leq \phi_1 |\xi_k|^\alpha + 2^{1-q_3} \frac{q_3}{\alpha} |\xi_i|^\alpha + \frac{1}{6} \sigma_k^\alpha + m_3 |\xi_k|^\alpha \quad (10)$$

with  $\eta_1 = (\psi_{1k})^{q_2}$ ,  $m_3 = \eta_1 \left( \frac{6\alpha}{\eta_1 q_3} \right)^{\frac{-q_3}{2-q_2}} \frac{2-q_2}{\alpha}$ , and  $\phi_1 = 2^{1-q_3} \frac{2-q_2}{\alpha}$ . Therefore, (9) is simplified as

$$|\xi_k|^{2-q_2} |H_k| \leq \phi_2 |\xi_k|^\alpha + \frac{1}{2} \sigma_k^\alpha + \phi_3 \sum_{l=1}^3 |\xi_l|^\alpha \quad (11)$$

with  $\phi_2 = 3b_J(\phi_1 + m_3)$  and  $\phi_3 = 2^{1-q_3} \frac{b_J q_3}{\alpha}$ . The virtual controller  $\tau_k^*$  is devised as  $\tau_k^* = -\xi_k^{q_3} \psi_{2k}$  and

$$\psi_{2k} = 1 + m_1 + n_k + \phi_2 + 3\phi_3 + h_2 \beta_1 \quad (12)$$

where  $h_2$  is a positive scalar such that  $h_2 \geq \tau_{\max}$ . Inserting inequalities (6) and (10) into  $\dot{V}_2$ , it arrives at

$$\dot{V}_2 \leq -\sum_{k=1}^3 h_2 \beta_1 \xi_k^\alpha + m_2 \sum_{k=1}^3 |\zeta_k|^\alpha - \sum_{k=1}^3 \left( h_1 \beta_1 + \frac{1}{4} \right) \sigma_k^\alpha \quad (13)$$

*Step 3:* A new Lyapunov candidate is selected here as  $V_3 = V_2 + \sum_{k=1}^3 \bar{Y}_k$  with  $\bar{Y}_k = \int_{\tau_k^*}^{\tau_k} (\delta^{q_3} - \tau_k^{*q_3})^{2-q_3} d\delta$ .

Using the properties of  $\bar{Y}_k$  from [9] and taking the time derivative of  $V_3$  yields

$$\dot{V}_3 \leq \dot{V}_2 + \sum_{k=1}^3 \zeta_k^{2-q_3} \left[ \tau_{\max} \mathbf{J}^{-1} \frac{\partial \tanh(\mathbf{u})}{\partial \mathbf{u}} \dot{\mathbf{u}} \right]_k - \sum_{k=1}^3 (2-q_3) \frac{d(\tau_k^{*q_3})}{dt} \int_{\tau_k^*}^{\tau_k} (\delta^{q_3} - \tau_k^{*q_3})^{1-q_3} d\delta \quad (14)$$

Due to the fact that  $\int_{\tau_k^*}^{\tau_k} (\delta^{q_3} - \tau_k^{*q_3})^{1-q_3} d\delta \leq |\zeta_k|^{1-q_3} |u_k - u_k^*| \leq 2^{1-q_3} |\zeta_k|$ . Defining  $\tilde{\psi}_{2k} = \psi_{2k}^{q_3}$ , one has

$$-\frac{d(\tau_k^{*q_3})}{dt} = \frac{d(\xi_k \tilde{\psi}_{2k})}{d\sigma_k} \frac{d\sigma_k}{dt} + \frac{d(\xi_k \tilde{\psi}_{2k})}{d\omega_k} \frac{d\omega_k}{dt} \quad (15)$$

Invoking (1), one obtains  $\frac{d\omega_k}{dt} = H_k + \tau_k$ . The inequality  $\tau_k = (\zeta_k + \tau_k^{*q_3})^{q_3} \leq (|\zeta_k|^{q_3} + |\xi_k|^{q_3})(1 + \psi_{2k})$  can be obtained by using the definition of  $\zeta_k$ . Employing the facts  $|\omega_k| \leq (|\xi_k|^{q_2} + |\sigma_k|^{q_2})(1 + \psi_{1k})$  and  $\omega_k^{q_2} = \xi_k^{1-q_2} + \omega_k^{*1-q_2}$ , one has

$$\frac{d(\xi_k \tilde{\psi}_{2k})}{d\omega_k} = \xi_k \frac{d\tilde{\psi}_{2k}}{d\omega_k} + \frac{\tilde{\psi}_{2k}}{q_2} (\xi_k^{1-q_2} + \sigma_k^{1-q_2} \psi_{1k}^{\frac{1-q_2}{q_2}}) \leq (\xi_k^{1-q_2} + \sigma_k^{1-q_2}) \underbrace{\left( \xi_k \frac{d\tilde{\psi}_{2k}}{d\omega_k} + \frac{\tilde{\psi}_{2k}}{q_2} \right)}_{\Theta_1} + \frac{\tilde{\psi}_{2k}}{q_2} \psi_{1k}^{\frac{1-q_2}{q_2}} \quad (16)$$

Recalling the inequality of  $H_k$  and assuming that for any  $k = 1, 2, 3$ , and  $1 < l < +\infty$ , the inequality  $3l(|\xi_k|^{q_3} + |\sigma_k|^{q_3}) \geq \sum_{i=1}^3 (|\xi_i|^{q_3} + |\sigma_i|^{q_3})$  holds, one has  $|H_k| \leq 3lb_J (|\xi_k|^{q_2} + |\sigma_k|^{q_2})(1 + \psi_{1k})$ . Using the property of  $T(\sigma)$  [1], i.e.,  $T(\sigma)T^T(\sigma) = \Phi^2 \mathbf{I}_3$  and substituting (16) into (15), one has

$$\begin{aligned} -\frac{d(\tau_k^{*q_3})}{dt} &= \frac{d(\xi_k \tilde{\psi}_{2k})}{d\sigma_k} \frac{d\sigma_k}{dt} + \frac{d(\xi_k \tilde{\psi}_{2k})}{d\omega_k} \frac{d\omega_k}{dt} = (|\xi_k|^{q_2} + |\sigma_k|^{q_2}) \Phi (1 + \psi_{1k}) \frac{d(\xi_k \tilde{\psi}_{2k})}{d\sigma_k} \\ &+ (\xi_k^{1-q_2} + \sigma_k^{1-q_2}) (|\zeta_k|^{q_3} + |\xi_k|^{q_3}) \Theta_1 (1 + \psi_{3k}) + (\xi_k^{1-q_2} + \sigma_k^{1-q_2}) (|\sigma_k|^{q_3} + |\xi_k|^{q_3}) 3lb_J \Theta_1 (1 + \psi_{1k}) \\ &\leq (|\sigma_k|^{q_2} + |\xi_k|^{q_2} + |\zeta_k|^{q_2}) \tilde{\phi} \end{aligned} \quad (17)$$

where  $\tilde{\phi} > 0$  can be obtained by using Lemma 1. Therefore, the last term of  $\dot{V}_3$  can be simplified as

$$-\sum_{k=1}^3 (2-q_3) \frac{d(\tau_k^{q_3})}{dt} \int_{\tau_k}^{\tau_k^*} (\delta^{q_3} - \tau_k^{q_3})^{1-q_3} d\delta \leq \sum_{k=1}^3 \frac{1}{4} |\sigma_k|^\alpha + \frac{1}{2} |\xi_k|^\alpha + \varphi_2 |\zeta_k|^\alpha \quad (18)$$

with  $\varphi_2 = \frac{1}{\alpha} \varphi_1^\alpha \left( \left( \frac{\alpha}{4q_2} \right)^{-q_2} + \left( \frac{\alpha}{2q_2} \right)^{-q_2} \right) + \varphi_1$  and  $\varphi_1 = 2^{1-q_3} (2-q_3) \tilde{\phi}$ . Substituting (13) and (18) into (14) yields

$$\dot{V}_3 \leq -\beta_1 \tau_{\max} \sum_{k=1}^3 (\sigma_k^\alpha + \xi_k^\alpha + \zeta_k^\alpha). \text{ Referring to [9], } V_3 \text{ can be simplified as } V_3 \leq 2 \sum_{k=1}^3 (\sigma_k^2 + \xi_k^2 + \zeta_k^2). \text{ Recalling}$$

Lemma 3,  $\dot{V}_3$  is simplified as

$$\dot{V}_3 \leq -\frac{1}{2} \beta_1 \tau_{\max} \left( 2 \sum_{k=1}^3 (\sigma_k^2 + \xi_k^2 + \zeta_k^2) \right)^{\frac{\alpha}{2}} \leq -\mathcal{G} \tau_{\max} V_3^\varrho \quad (19)$$

with  $\mathcal{G} = 2^{-\varrho} \beta_1$  and  $\varrho = 0.5\alpha$ . Using the lemma 1 in [4], one can prove that the closed-loop system is stable within a finite time  $T_s$ . It is bounded by  $T_s \leq \frac{1}{\tau_{\max}} \frac{1}{\mathcal{G}(1-\varrho)} V_3(0)^{1-\varrho}$ . The larger the value of  $\tau_{\max}$ , the smaller  $T_s$ . Therefore, the reported control method reveals the inverse proportional relationship between the input constraint value and the system's convergence time. The Theorem 1 is thus proved.

## 4. Simulation Results

Numerical examples are conducted in this section to illustrate the presented theoretical analysis. In simulation, the moment of inertia of the considered satellite is  $\mathbf{J} = [20 \ 0 \ 0.9; 0 \ 17 \ 0; 0.9 \ 0 \ 15] \text{ kg} \cdot \text{m}^2$ . The initial attitude of the satellite is  $\sigma = [0.25, 0.31, -0.24]^T$ , and the initial angular velocity is  $\omega = [0, 0, 0]^T \text{ rad/sec}$ . The main gains of the controller are  $\mu = 102/721$ ,  $\lambda = 4$ ,  $\beta_1 = 1.3$ , and  $h_i = 1$  with  $i = 1, 2$ .

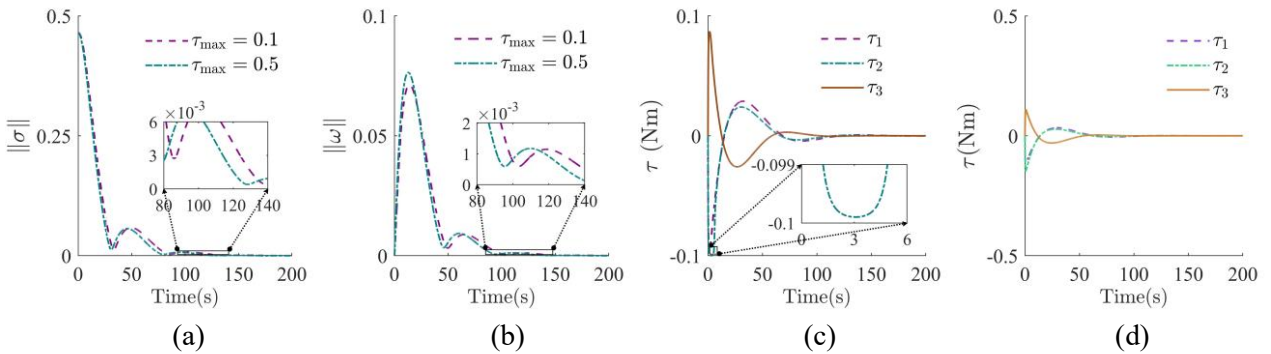


Fig. 1: Comparative simulation results. (a) Time response of  $\|\sigma\|$  for different  $\tau_{\max}$ ; (b) Time response of  $\|\omega\|$  for different  $\tau_{\max}$ ; (c) Time response of  $\tau$  under  $\tau_{\max} = 0.1$ ; (d) Time response of  $\tau$  under  $\tau_{\max} = 0.5$ .

Employing the finite-time controller for achieving required attitude maneuver, the compared simulation studies with  $\tau_{\max} = 0.5 \text{ Nm}$  and  $\tau_{\max} = 0.1 \text{ Nm}$  are implemented to analyze the control performance of the satellite attitude. The simulation results are presented in Figs. 1 (a)-(b). The attitude and the angular velocity of the considered satellite are ensured to be finite-time stable. Specifically, the norms of the errors  $\|\sigma\|$  and  $\|\omega\|$  are maintained below the thresholds of  $\|\sigma\| \leq 6 \times 10^{-3}$  and  $\|\omega\| \leq 2 \times 10^{-3}$ . It is obvious that when  $\tau_{\max} = 0.5 \text{ Nm}$ , the convergence rate of the attitude stabilization is faster than that when  $\tau_{\max} = 0.1 \text{ Nm}$ , and it also has a higher control accuracy. It confirms the conclusion that the attitude convergence time is inversely proportional to the input constraint value. The corresponding commanded control input is shown in Figs. 1 (c)-(d). The commanded input signals are always within the input constants.

## 5. Conclusion

This paper has proposed a new finite-time control scheme for satellite attitude control systems with input constraints by using the adding power integrator methodology. It has demonstrated that the convergence time

of the state trajectory is inversely proportional the input constraint. By introducing a bounded hyperbolic tangent function to augment the control variables, the attitude control system is expanded as a higher-order system, thereby solving the input constraint issue. With the help of the adding power integrator-based controller, the finite-time stability for the attitude control system is achieved.

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## 7. References

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