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Decision Making with Iterative Game under the Semi-Perfect Information

Sanghyuk Lee [1] and Azizbek Marakhimov [2+]

[1] Department of Computer Science, New Uzbekistan University, Uzbekistan

[2] Department of Industrial Management, New Uzbekistan University, Uzbekistan

Abstract. Decision making framework with an iterative game structure has been proposed. In the proposed structure, the maximum benefit for each player is resolved by the iteration process. Based on the payoff matrix, the optimal solution is sought by comparing it with the criterion set by the players. To maximize the benefit for each party (buyer and seller), an iterative game structure is proposed based on the given payoff matrix and iterative machine. For real-world application, practical example is considered, and a feasible solution is obtained. In comparison with the existing body of research on game theory under semi-perfect information, the provided solution is far from their payoff but the result would be acceptable for two parties.

Keywords: Nash equilibrium, Iterative Game, Payoff Matrix, Benefit Maximizing

1. Introduction

Existing research on decision making has been applied to diverse areas such as business management, engineering, and social science [1, 2]. To obtain relevant results, more rational measures are needed, thus research to design such measures has been conducted by the numerous authors [3, 4]. Resolutions to deal with purchasing, settling, and fixing in agreements require negotiation and communication [5, 6].

In this regard, numerous case studies have focused on the said process [7]. In this research, an iterative game structure is proposed and compared to other methods to resolve the problem. As noted, game theory has been widely used to find equilibrium solution between conflicting interests from different parties. With the game theoretic approach, the iterative game structure is useful to reach an agreement within a tolerance bound. This is clarified in the signal flow chart. Based on the payoff matrix, a semi-perfect information structure is proposed with a machine [7]. With the help of repeated game structure, we propose an iterative game structure. Under the multi-criteria decision problem, it does not have a fixed period due to the number of criteria. Thus, it is necessary to set the termination condition in advance. The proposed approach has the following advantage to solve the existing problems; addressing termination failure in agreement with the limited utility payoff. On the other hand, we emphasize the design of decision measures in the middle of iteration by comparison with the payoff.

To verify the effectiveness of the proposed method, a more practical example is also considered: purchasing cars from the brand viewpoint of the buyer and benefit maximization from agent (seller) viewpoint. This approach was proposed from the game theoretic knowledge under perfect information. The iterative process is explained based on a machine structure for both players (buyer and seller) to maximize their benefits. A feasible solution is obtained by the machine processing. The result is far from the Nash equilibrium because *social norm* should be considered even when two players keep the rational and logical strategies.

The paper is organized as follows: Section 2 provides a brief introduction to game theory and its iterative structure, including a proposal for applying the iterative structure on the example. Based on the practical example, iterative procedure is outlined and the termination condition proposed in Section 3. Illustrative example is presented in Section 4, and solved using the proposed iterative structure. Calculation results show the maximizing payoff for each buyer and seller under the given payoff matrix. Conclusions are presented in Section 5.

+ Corresponding author.
E-mail address: a.marakhimov@newuu.uz.

2. Preliminaries

In this section, the concepts of game theory and repeated structure are briefly illustrated. Additionally, the decision sequence is illustrated with the information flow graph.

2.1. Game Theory

Game theory was initiated in 1944 by John Von Neumann and Oscar Morgenstern [8]. And it influenced to the economy, politics, and administration decision making area [4 -6]. And the non-zero sum game, the cooperative game was later introduced by John Nash in 1950 [9]. Later, the field has extended to include Quantum game, Fuzzy game, Evolutionary game, and others [7]. Game theory provides a knowledge background to explain natural and complex systems, and has been also applied to industry and to the social consensus and control problem in engineering [5,6].

To construct iterative structure, it is needed to survey the example of Prisoner's dilemma. Originally, it includes players' rational mind, and their decision converge to the Nash equilibrium [7]. In prisoner's dilemma, two suspects involved in a major crime are held in separate cells. There is enough evidence to convict both suspects of a minor offence, but not enough evidence to convict either of them of the major crime unless one of them decide to act as an informer against the other (finking). From the Table 1, it is

Table 1: Two alternatives and four criteria

		Suspect 2	
		Quiet (**C**)	Fink (**D**)
Suspect 1	Quiet (**C**)	3, 3	0, 4
	Fink (**D**)	4, 0	1, 1

From the utility matrix, it is obvious that Suspect 1's and 2's ordering from best to worst with numeric markings are;

$\boldsymbol{u_1}\{\text{Fink, Quiet}\} = 4 > \boldsymbol{u_1}\{\text{Quiet, Quiet}\} = 3 > \boldsymbol{u_1}\{\text{Fink, Fink}\} = 1 > \boldsymbol{u_1}\{\text{Quiet, Fink}\} = 0,$
$\boldsymbol{u_2}\{\text{Quiet, Fink}\} = 4 > \boldsymbol{u_2}\{\text{Quiet, Quiet}\} = 3 > \boldsymbol{u_2}\{\text{Fink, Fink}\} = 1 > \boldsymbol{u_2}\{\text{Fink, Quiet}\} = 0.$

In the payoff matrix in Table 1, both suspects have a high payoff when they choose to betray each other. However, if they cooperate, both parties enjoy a reliable payoff without any possible low payoff with Nash equilibrium: (1, 1).

2.2. Iterative Game Structure

Generally, a decision can be made after repetitive negotiation. Hence, it is required to consider an iterative structure, as we consider in Fig. 1. Our concern is to maximize mutual benefit. From the Fig. 1, iteration can be terminated by the number of iterations or payoff bound to be set in advance. Hence, it is possible to pick up the most valuable payoff value to each player. However, an infinite repeated game depends on the number of criteria, and it is necessary to set boundary values for each player in advance.

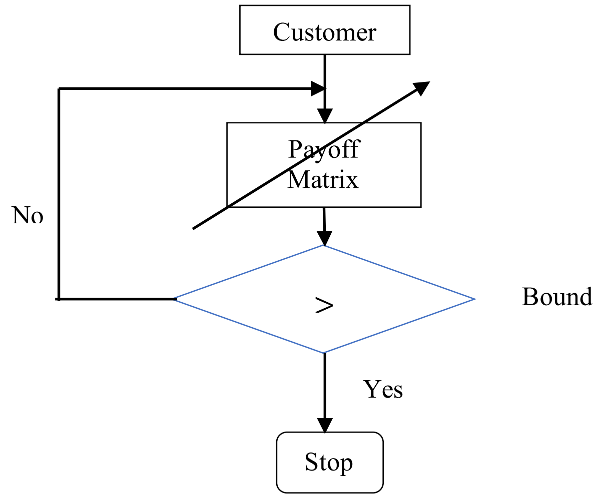

Fig. 1: Decision process with flow chart

3. Repeated Game Structure

An iterative game structure is proposed by considering an iterative point of view. Modified structure is organized with machine for each player, and it is explained through the Prisoner's Dilemma.

3.1. Iterative Structure with Machine

The goal of an iterative game is to isolate types of strategies that support mutually desirable outcomes within the game structure. When repeated, this structure may be interpreted in terms of a *social norm* [7]. Eventually, it is necessary to sustain mutually desirable outcomes. Socially desirable outcomes cannot be sustained if players are short-sighted but can be sustained if the players have long-term objectives. Furthermore, the set of equilibrium outcomes of a repeated game is so large that the notion of equilibrium lacks predictive power.

We begin by defining a machine, a function $f_i: Q_i \rightarrow A_i$ and a transition function $\tau_i: Q_i \times A \rightarrow Q_i$. Where, Q_i, A_i and A denote the set of states, an action of every state, and action profile, respectively. The transition function is composed by taking inputs as the current state and the list of actions chosen by the other players. This fits the natural description of a *rule of behavior* or *strategy* as a plan of how to behave in all possible circumstances. Machine structure was proposed by Osborne in his book [7]. Its repeated structure depends on Machine for each player, as in the Prisoner's Dilemma. Short descriptions are as follows:

Machine 1 (Player 1):

- If P_2 keeps Quiet ***(C)***, P_1 keeps Quite; cooperation of both being Quite guarantees a win-win situation.
- If P_2 changes to Fink ***(D)***, P_1 changes to Fink; this moves shifts to Nash equilibrium.
- If P_2 keeps Fink, P_1 keeps Fink; they stay in Nash equilibrium.
- If P_2 changes to Quiet, P_1 keeps Fink; P_1 maximizes his payoff.

This machine for player 2 in the prisoner's dilemma.
Machine 2 (Player 2):

- P_2 starts by playing Quiet but switches to Fink if P_1 chooses to stay Quiet.
- P_2 returns to Quiet only if P_1 chooses to Fink.
- If P_1 chooses to Fink, P_2 keeps Quiet.
- If P_1 chooses to stay Quiet, P_2 switches to Fink.

From the outcomes of the repeated prisoner's dilemma, a total of six periods of iteration were accomplished to reach the most beneficial outcome for each party [7]. However, in the context of a multi-criteria problems as described below, such periods of iteration may not exist.

3.2. Machine Structure on Multi-criteria

The main idea of the repeated games is to find a mutually desirable outcome such as (C, C) scenario in the prisoner's dilemma, which occurs in every period. However, when it is a multi-criteria problem, Agent and Buyer have multiple criteria and attributes with distinct payoff values. For example, consider a situation when the buyer evaluates three attributes and the agent considers four criteria, as illustrated in Table 2, with respective payoff values: (b_i, c_j). In this case, both buyer and agent haver their own decision limits denoted as b_{i_lim} and c_{j_lim}. Decision limit values should be considered within the repeated period.

Table 2: Two alternatives and four criteria

		Agent			
		Benefit 1	Benefit 2	Benefit 3	Benefit 4
Buyer	**A**	(b_1, c_1)	(b_1, c_2)	(b_1, c_3)	(b_1, c_4)
	B	(b_2, c_1)	(b_2, c_2)	(b_2, c_3)	(b_2, c_4)
	C	(b_3, c_1)	(b_3, c_2)	(b_3, c_3)	(b_3, c_4)

From the buyer's perspective, he/she needs to obtain payoff information through product details such as b_1, b_2 and b_3. The agent recommends the product without revealing his/her payoff values to the customer: c_1, c_2, c_2 and c_4. To generalize the interaction between the Agent and the Buyer, consider the following machine:

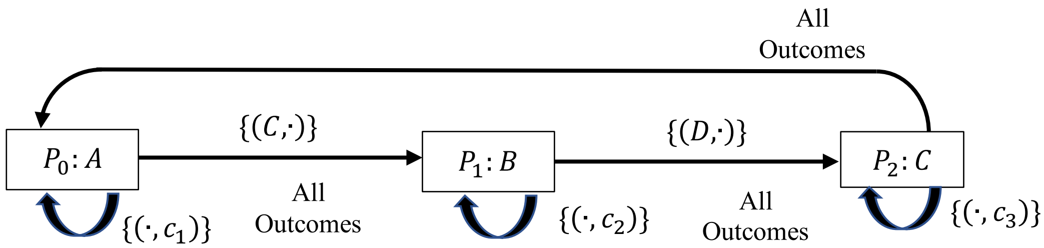

Fig.2: The machine M_1 for Buyer

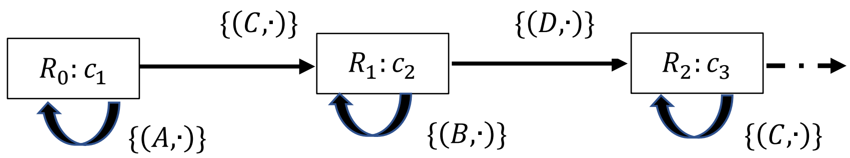

Fig.3: The machine M_2 for Agent

Fig. 2 and Fig. 3 illustrate two machines for Player 1 and Player 2, searching for the payoff follow the arrows in the Table 2. Therefore, machines as illustrated in Fig. 2 and Fig. 3 are necessary. The number of states are different from the prisoner's dilemma, featuring 4 and 2 states across a period of 6. With the machine structure as in Fig. 2 and Fig. 3, there is no fixed period for multi-criteria problem: it depends on the number of criteria.

3.3. Termination Strategies

Decision process can be terminated under the following conditions;

- The maximum value of $u(b_{i\_lim})$ among the buyer's b_i options during the period is achieved.
- The agent's benefit does not meet his/her $u(c_{j\_lim})$, requiring the recommendation of another option to the customer.

- The buyer can terminate the process regardless of the cost. In this case, it is rather an emotional decision.

Iterative outcomes do not simply replicate the Nash equilibria of the constituent game. To support such outcome, each player must be deterred from deviating by threat of being *punished*. Such punishment may take many forms. A common possibility is that each player uses a "trigger strategy": any deviation causes him/her to carry out a punitive action that lasts indefinitely. In the equilibria that we study in this section, each player adopts such strategy. Based on the different vector forms of the iterated function systems (IFSs), two novel score functions are proposed. Their properties are further illustrated in this subsection.

4. Illustrative Examples

Basically, the agent uses specific strategies to sell the product, which involve ordering criteria to maximize his/her benefits: if the customer refuses based on the first criterion, the agent moves to the second criterion to sell the product. This approach can be illustrated by the payoff matrix, using the example of a buyer interested in buying a car from one of three brands – KIA, BMW, and Toyota. From the buyer's perspective, payoff value is flexible and can be adjusted as: whether he/she emphasizing cost, utility, convenience, or sustainability. To align with the buyer's requirements, agent's suggestions should reflect the buyer's viewpoint. The considered payoff matrix can be generalized to a decision-making problem by adjusting the priority of the criteria. However, setting the payoff for each attribute presents a challenge.

Table 3: Three alternatives and four criteria buying car

		Agent			
		Priority 1	Priority 2	Priority 3	Priority 4
Buyer	**Toyota**	(2, 2)	(2, 2)	(3, 3)	(3, 4)
	BMW	(3, 2)	(3, 2)	(3, 3)	(3, 3)
	KIA	(1, 2)	(1, 3)	(1, 3)	(1, 4)

From the machines shown in Fig. 2 and Fig. 3, a transaction with BMW is indicated in red box with the payoff set as (2.5, 2). However, Toyota is chosen with the payoff set as (2.5, 2.5).

5. Conclusions

In this paper, an iterative game structure is proposed to resolve multi-criteria decision making. To establish this structure, the game theory concept of the Prisoner's Dilemma is utilized. Payoff values are assumed based on the preference values. It is suggested that proper measures for valuing the attributes should be derived from the existing results or further developments. The paper emphasized the organization of an iterative game structure. The state machine considered in the existing repeated game incorporates a process period for the prisoner's dilemma. However, for the multi-criteria decision making, it is represented as a non-periodic structure, hence it requires termination conditions. In the provided example, we consider a car purchasing model with multi-criteria, and termination cases are evaluated with the predefined payoff boundaries.

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7. References

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