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# Quad-rotors Unmanned Aerial Vehicle Stability Augmentation Model Reference Adaptive Control

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**Abstract.** The Quad-Rotors UAV is a complex nonlinear under actuated system. It may afflict with the wind disturbance, sensor noise and model uncertainty exists when flying outdoor. These problems bring many difficulties to the controller design. For the real system, there are lots of constraints to the control quantity output, such as the driven capability of the actuator. In this paper, a new method which is called Stability Augmentation Model Reference Adaptive Control (SAMRAC) is proposed to deal with these difficulties. Firstly, by analyzing and simplifying the dynamic model, we attribute the system to a double integrator characteristic. Using the stability augmentation module, the poly of the system can be stabilized in the stable area in the complex plane. In order to insure the tracking performance of the UAV, the model reference adaptive control method has been introduced. The simulation results proved that this control strategy is more robust and effective on rejection of uncertainty and external disturbance contrary to the normal PID controller. Especially, the control quantity output is much small and smooth.

**Key words**: The Quad-Rotors UAV, Stability Augmentation Model Reference Adaptive Control

## 1. Introduction

In recent years, UAV has attracted great academic interest for its potential industrial and military applications. Unmanned aerial vehicles (UAVs) can fly independently under extreme conditions and are not affected by emotions of pilots [1]. Therefore, UAVs are suitable for investigating or performing special tasks. At the same time, UAVs have great application value in many fields, such as completing search and rescue, building survey, geological exploration, and security and reconnaissance tasks[2].

At present, UAVs can be roughly divided into two categories: fixed-wing UAVs and rotor UAVs. Rotor UAV has attracted wide attention because of its small size, small land area, and the ability to achieve vertical take off and landing, hovering in the air and other actions that fixed-wing UAV is difficult to accomplish. Quad-rotor UAV is a kind of under-actuated coupled nonlinear system. In view of the characteristics of Quad-rotor UAV, the researchers of Clemson University in the United States [3] have given robust trajectory tracking control for the parameter uncertainties of the under-actuated Quad-rotor UAV model. In this model, Lagrange modeling method is used, and then the uncertainties are reduced to additive uncertainties. For Quad-rotor unmanned helicopter, the commonly used control methods include: PID, sliding mode control [4], back stepping [5] feedback linearization and so on. Great achievements have been made in the research of control methods for Quad-rotor UAV. However, there are few studies on the control methods from the perspective of practical application [6, 7].

In the research of Quad-rotor UAV control algorithm, most of them rely on the mathematical model of the system, and the control structure is complex, so it is not easy to realize. At present, the research of UAV outdoor autonomous flight control is still in its infancy. Aiming at the external disturbances, uncertainties in the modeling process and output constraints of the controller in the outdoor flight of a Quad-rotor UAV, a

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model reference adaptive control for augmented stability is proposed, and good experimental results are obtained.

## 2. Model Description

The structure of the Quad-rotor UAV is shown in Figure 1, in which the front and rear motors rotate counter clock wise and the left and right motors rotate clockwise. The position and attitude of aircraft are changed by changing the speed of Quad motors. The dynamic model of Quad-rotor UAV is established by Newton-Euler equation as shown in Figure 1.

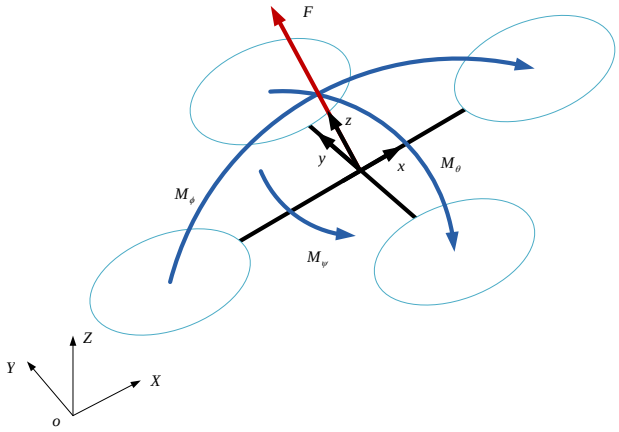


Fig. 1: Structural schematic of a Quad-rotor UAV

The transformation matrix from Euler angle to body rotation speed is formula (1) and (2). Among them, $M_\phi$、$M_\theta$、$M_\Psi$ is pitch, roll and yaw moments are the key research issues, and the aerodynamic characteristics of the rotor are not considered in detail here. Considering the general conclusion of rotor dynamics and the rationality of the actual system, two hypotheses are proposed here.

1） The force and moment produced by the rotor in the process of rotation are linearly related to the square of the rotor speed. That is $F=k_F\omega^2, T=k_T\omega^2$. The linear relationship between rotor force, moment and speed is satisfied.

2） Motor speed can accurately follow the given signal.

Based on the above two assumptions, the dynamic model of Quad-rotor UAV is given as follows:

$$\begin{cases} \ddot{X}=\left(\sin\psi\sin\phi+\cos\psi\sin\theta\cos\phi\right)F/m \\ \ddot{Y}=\left(-\cos\psi\sin\phi+\sin\psi\sin\theta\cos\phi\right)F/m \\ \ddot{Z}=-g+\left(\cos\theta\cos\phi\right)F/m \\ \dot{p}=\dfrac{I_y-I_z}{I_x}qr-\dfrac{J_{TP}}{I_x}q\left(-\Omega_1+\Omega_2-\Omega_3+\Omega_4\right)+M_\phi \\ \dot{q}=\dfrac{I_z-I_x}{I_y}pr+\dfrac{J_{TP}}{I_y}p\left(-\Omega_1+\Omega_2-\Omega_3+\Omega_4\right)+M_\theta \\ \dot{r}=\dfrac{I_x-I_y}{I_z}pq+M_\psi \end{cases} \tag{1}$$

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}1 & \sin\phi\tan\theta & \cos\phi\tan\theta\\ 0 & \cos\phi & -\sin\phi\\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta\end{bmatrix}\begin{bmatrix}p\\ q\\ r\end{bmatrix} \tag{2}$$

Where $X$、$Y$、$Z$ represent the position of the aircraft in the inertial coordinates; $\Phi$、$\theta$、$\varphi$ represent the aircraft; $p$、$q$ and r $Z$ represent the angular velocity of the aircraft rotating around three axes in the body coordinate system; $J_{TP}$ is rotating inertia around rotating axis for rotor motor system. $I_{XX}$、$I_{YY}$、$I_{ZZ}$ are the moment of inertia of an aircraft around the $x$、$y$、$z$ axis in the body coordinate system. $k_1$ is the lift coefficient of the rotor, $k_2$ is the drag coefficient of the rotor, $m$ is the mass of the aircraft, $l$ is the distance from the rotor to the geometric canter of the aircraft, $\Omega_1$、$\Omega_2$、$\Omega_3$、$\Omega_4$ are the angular velocities of the Quad rotors of the Quad-rotor unmanned aerial vehicle.

## 3. Design of Stability Augmentation Model Reference Adaptive Control

The space trajectory tracking control of UAV mainly considers three-dimensional motion in geodetic coordinate system. For convenience of study, the attitude of UAV is assumed to be stable. This paper mainly studies the trajectory tracking control method of UAV in space. In order to ensure that the tracking error and overshoot of UAV are small in the course of trajectory tracking, the model reference adaptive control is introduced to make the UAV maintain certain tracking performance under the uncertainties of its own parameters and external disturbances (such as wind disturbances).

Because the UAV has symmetry in the $x$ and $y$ directions, we take the $x$ direction as an example. The simplified $x$-direction dynamic model is $\|F\| < \xi$, $\xi$ is a real number less than infinite.

$$\ddot{X} = \frac{K}{m}\cos\phi \cdot F + \Delta, \quad \Delta = \sin\psi \sin\phi \cdot F \tag{3}$$

Because $F$ is bounded, $\Delta$ is bounded. Therefore, the simplified forward flight model is:

$$\ddot{X} = \frac{K}{m}F' \tag{4}$$

Where $F' = \cos\phi \cdot F$. Then the model is simplified to a double integral system. The system has poles on the imaginary axis. It's not the smallest phase. Therefore, the stability-increasing inner ring [1] is considered. The forms of stabilization controller are as follows:

$$U_{SAC}(t) = \Pi \frac{A(p)}{B(p)} \tag{5}$$

where $A(p) = p^2 + \lambda_1 p + \lambda_0$, $B(p) = p^2 + b_1 p + b_0$, $B(p)$ is a stable polynomial. $\Pi$ is the control gain. Then the generalized object can be expressed as:

$$x(t) = \Pi \frac{K}{mp^2} \frac{p^2 + \lambda_1 p + \lambda_0}{p^2 + b_1 p + b_0} u(t) = P_a u(t) \tag{6}$$

$$P_a = \Pi \frac{K}{mp^2} \frac{p^2 + \lambda_1 p + \lambda_0}{p^2 + b_1 p + b_0} \tag{7}$$

Where $m$ the mass of UAV and $K$ is a constant. In order to configure the system into a typical second-order system, let $\lambda_1 = \lambda_2 = 0$, In order to ensure the fast response and steady-state performance, the damping coefficients $\xi = 0.25$ can be used to solve $b_1 = 0$ and $b_0 = 0.5$. The parameters $\Pi$ can be optimized according to the required output performance indices, and finally $\Pi = -0.0639$ can be obtained.

The stabilized inner ring can stabilize the controlled object in the left half plane of the complex plane. Thus, the design requirement of model reference adaptive controller is satisfied. Model Reference Adaptive Control System (MRACS) mainly studies the case that the model of the controlled object of the system is unknown or the model contains unknown parameters. This control method generally requires that the controlled system be a steady system with minimum phase.

If the order of the object model is $n_a$ and the molecular order is $m_a$, then the relative order is $n_a - m_a$. If the denominator order of the reference model is $n_b$ and the molecular order is $m_b$, the relative order of the reference model is $n_b - m_b$. According to the causality of the system, it is necessary to satisfy $n_b - m_b > n_a - m_a$, that is, the relative order of the reference model is larger than that of the controlled object. The polynomials $R(p)$ and $H(p)$ satisfying the Diophantine equation are selected, where $p$ represents the differential operator. Here $n_b = 2$, $m_b = 0$, $l = 0$, $\rho = 0$. The reference model is as follows:

$$P_{ref} = \frac{0.072}{p^2 - 1.11p + 0.182} \tag{8}$$

Let $Q(q) = g_2 q^2 + g_1 q + g_0$, $D(q) = d_0$. The output of the controlled object is:

$$y(k) = \frac{1}{A_m(q)}\left[\left(g_2 q^2 + g_1 q + g_0\right)u(k) + d_0 y(k)\right] \tag{9}$$

The estimated parameters are:

$$\Theta^T = \begin{pmatrix} g_0 & g_1 & g_2 & d_0 \end{pmatrix} \tag{10}$$

The signal vector is:

$$\xi^T(k)=\begin{pmatrix}u(k) & qu(k) & q^2u(k) & y(k)\end{pmatrix} \tag{11}$$

The state filter vector is:

$$\zeta(k-2)=\frac{1}{p^2+2.67p+3.56}\xi(k-2) \tag{12}$$

From Formula (12): $\zeta(k)=\xi(k-2)-2.67\zeta(k-1)-3.56\zeta(k-2)$

The estimated output of the controlled system is: $\hat{y}(k)=\hat{\Theta}^T(k-1)\zeta(k)$

Object input is:

$$u(k)=\frac{1}{\hat{\theta}_1}\left[b_{m0}u_m(k-2)-\sum_{i=2}^{4}\hat{\theta}_i\xi(k-2)\right] \tag{13}$$

Parametric mediation rate:

$$\hat{\Theta}(k)=\hat{\Theta}(k-1)-\frac{\Gamma(k-1)\zeta(k-m-\rho-2)\varepsilon(k)}{1+\zeta^T(k-m-\rho-2)\Gamma(k-1)\zeta(k-m-\rho-2)} \tag{14}$$

$$\Gamma(k)=\Gamma(k-1)-\frac{\Gamma(k-1)\zeta(k-m-\rho-2)\zeta^T(k-m-\rho-2)\Gamma(k-1)}{1+\zeta^T(k-m-\rho-2)\Gamma(k-1)\zeta(k-m-\rho-2)} \tag{15}$$

Where $\Gamma$ is a symmetric positive definite gain matrix. So far, the stabilization adaptive trajectory tracking controller for Quad-rotor UAV has been designed.

## 4. Simulation Analysis

According to the control strategy and decoupling method mentioned above, the flight situation of Quad-rotor UAV is simulated and analysed. Fig. 6 and 7 are the forward flight trajectory tracking curves of UAV under the traditional PID controller and the augmented model reference adaptive controller respectively, considering the parameter perturbation.

The actual data of UAVs are as follows: (the following data are all in the International Unit System):

$m=1.12$, $g=9.8$, $I_x=0.1$, $I_y=0.1$, $I_z=0.14$, $l=0.6$, $k_1=4.12\times10^{-5}$, $k_2=3.54\times10^{-9}$, and $I_r=1.5\times10^{-4}$.

Fig. 2 shows that both controllers can stabilize the system without external disturbance, unmodeled dynamics and sensor noise, but the time of overshoot and overshoot adjustment under PID control is long. The overshoot and small adjustment time of the system is short and the steady state is error-free under the augmented model reference adaptive.

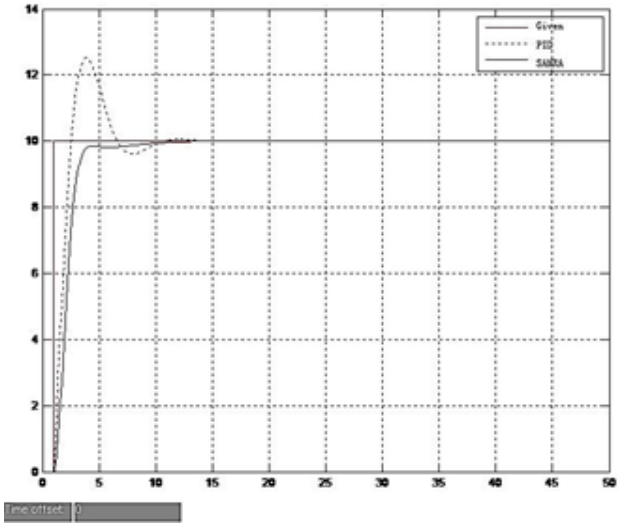


Fig. 2: The trajectory tracking curve of UAV controlled by PID controller and augmented model reference adaptive control with step input

Fig. 3 is a response curve considering the unmodeled dynamics of the system, i.e. the uncertainty $\Delta$ shown above. It can be seen that the steady-state error of the system under the PID controller is large, while the overshoot of SAMRAC is small and the steady-state error is small.

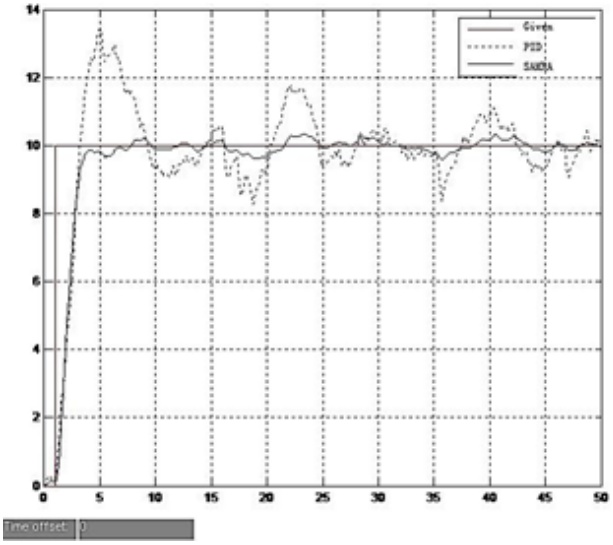


Fig. 3: Trajectory tracking curves under two controllers considering unmodeled dynamics

Figure 4 shows the UAV output curve considering the presence of sensor measurement noise. It can also be seen that the tracking performance of SAMRAC is better.

Fig. 5 shows the output of control variables under two kinds of controllers. It can be seen from the graph that even if the output effect of SAMRAC is equal to that of PID with better parameters, the output of control variable is drastic and the peak value is large, which is very disadvantageous to the actuator. It may lead to improper response of the actuator, and even cause excessive wear and tear of the actuator, which greatly reduces the service life of the actuator.

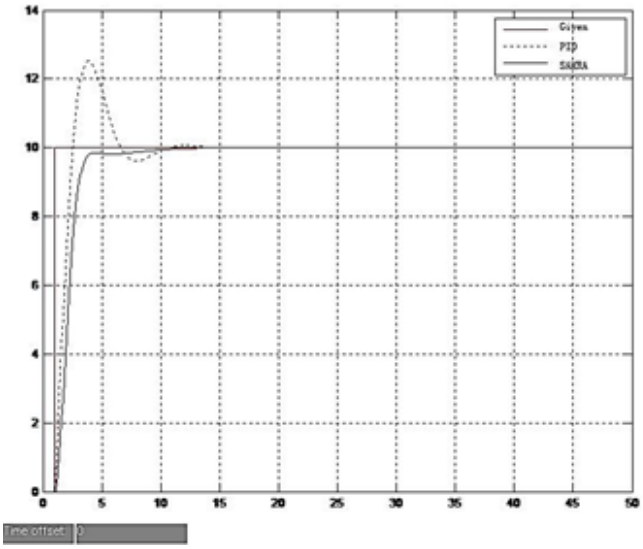


Fig. 4: The horizontal position output curve of the system with sensor noise and unmodeled dynamics

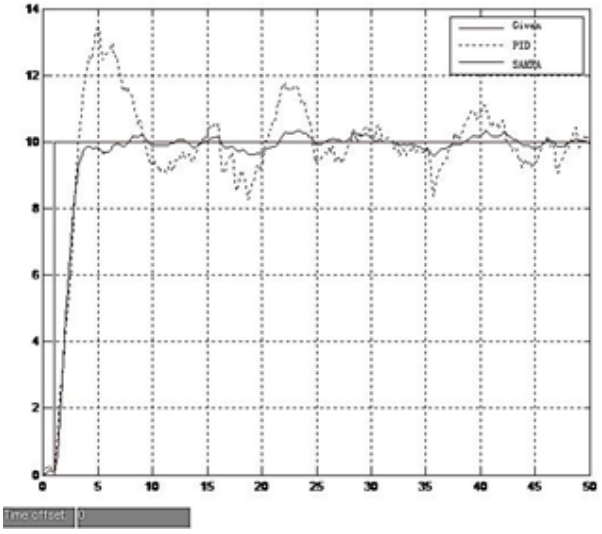


Fig. 5: Trajectory tracking curve of the system with parameter uncertainties under the PID controller

Figures 6 and 7 show the comparison of output curves between the two controllers when uncertain parameters exist in the system. It is easy to see that SAMRAC controller has a good resistance to the change of uncertain parameters.

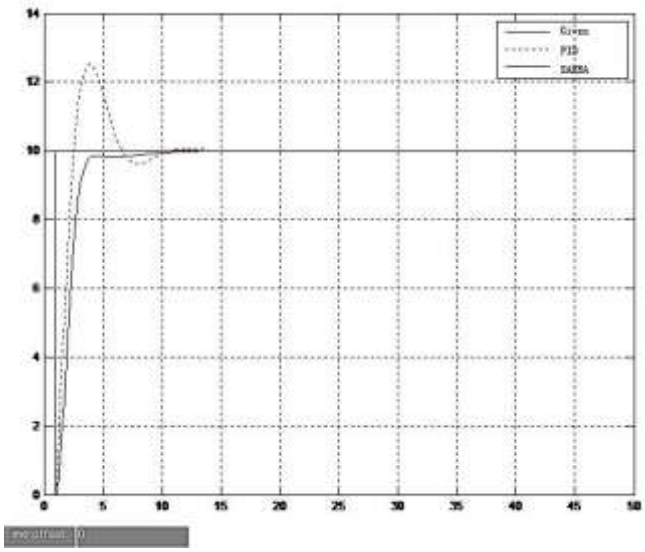

Fig. 6: Trajectory tracking curve of the system with parameter uncertainties under the PID controller

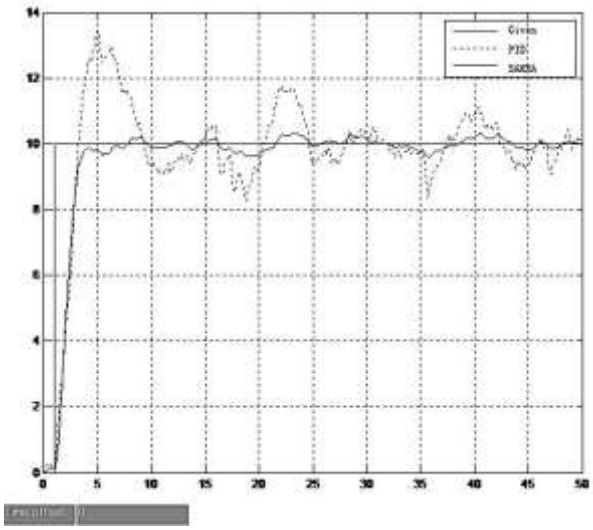

Fig. 7: Trajectory tracking curve of the system with parameter uncertainties under augmented stability model reference adaptive

## 5. Summary

In this paper, an augmented model reference adaptive control (SAMRAC) is proposed for UAV outdoor autonomous trajectory tracking due to wind disturbance and unmodeled dynamic uncertain parameters. The advantage of this controller is that it can be applied to non-minimum phase control. The simulation results show that the controller has good resistance to unmodeled dynamics and external disturbances. Compared with traditional PID controller SAMRAC controller, it has high steady-state accuracy, short transition process and small overshoot. Most of the engineering significance is that SAMRAC controller control output is small, it is not easy to saturate the actuator, and control output changes are not drastic, which is conducive to ensuring the long-term stable operation of the actuator, while ensuring low power consumption of the system. In summary, the model reference adaptive control of augmented stability has good control effect for the nonlinear uncertain system.

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