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Study on Selection of Dependent Variables of Relief Amplitude Optimum Statistical Unit in China based on GIS

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Abstract. The analysis of relief amplitude has great significance for regional geomorphic classification and geological hazard risk assessment. The calculation of relief amplitude lies in the choice of statistical unit. However, the definition of optimum statistical unit is fuzzy, the accuracy and applicability of dependent variables and fitting model remain to be discussed so far. Therefore, taking SRTM-DEM data for example, this paper extracted relief amplitude in China under different grid units using ArcGIS software, respectively discussed in detail the following three aspects from mathematical model of relief amplitude: mathematical definition of relief amplitude, research ideas of optimum statistical unit, selection of dependent variables and fitting model. Research shows that: The research idea at present is different from the idea first put forward, so the corresponding calculation model required correction; We should looking for a stable point when using artificial drawing method to determine optimum statistical unit; The result of using "power model" to fit the highest frequency of relief amplitude is the most suitable, and maximum relief amplitude is not suitable for the research of nationwide optimum statistical unit as a dependent variable.

Keywords: relief amplitude, optimum statistical unit, China, SRTM-DEM, GIS

1. Introduction

Relief amplitude is a significant index that describes topographic feature and reflects the change of topographic relief. It is commonly shown by the elevation difference between the highest point and the lowest points in a certain area [1]. Moreover, relief amplitude is a result of interaction between tectonism and denudation, which is widely used in research on evolution characteristics of orogenic belt and plateau mountains, and regional soil erosion assessment. It is also an object basis for comparative study of regional topographic and classification of physiognomy. According to some researches, there is a prominent correlation between relief amplitude and landslide [2]. The key to calculating relief amplitude is the selection of statistical units. Area sizes can reflect the integrity of mountains, which is universality. Determining the optimum statistical unit is the core step of relief amplitude calculation.

According to the existing researches on the extraction of optimum statistical unit, there are several issues that need to be discussed further in this paper:

1) "Inflection point": in the existing researches, the definition of inflection point in physiognomy is different from the definition in mathematics, however, based on references [1] we think that

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inflection point in physiognomy is the same point in mathematics. And how to select the optimum statistical unit point need to be discussed further.

2) Research idea: in the earliest studies [1,3,4], scholars use the highest frequency unit area as the optimum statistical unit based on the change rule of a single point in different unit area. Nevertheless, in the later references scholars always calculated relief amplitude for all grid cells (sample points) in a unit area first, and then studied on the change rule of relief amplitude in different unit area. The difference between these two research ideas have not been studied yet.
3) Dependent variable selection: in the selection of dependent variable, some scholars choose maximum relief amplitude [5], and some scholars use average relief amplitude[6]. However, there is no research on the accuracy and applicability of different dependent variable yet.
4) Fitting model selection: mathematical definition of relief amplitude points out that the change rule of amplitude conforms to logistic regression model. However, in this paper amplitude parameters extracted from the second idea do not meet this rule.

Thus, based on SRTM-DEM data, this paper using ArcGIS mainly studies relief amplitude on three aspects: mathematical definition, research idea of the optimum statistical unit and selection of dependent variable and fitting model, to provide some references for electric power construction, geological disasters, environment protection and ecological protection.

2. Mathematical Concept and Simulation Equation of Relief Amplitude

2.1. Mathematical Concept

Tu Han-ming [1] is the first person who proposes the mathematical model of relief amplitude for China relief amplitude research. In order to discuss the mathematical meaning of physiognomy “inflection point”, this paper study further based on Tu’s researches which points out that relief amplitude is expressed as:

$$R_a = q \cdot h \cdot \Delta h/\Delta H = q \cdot h \cdot (1 - h/\Delta H) \tag{2-1}$$

where q is morphological parameter of geomorphic system, h is relief height, Δh is relative height, and ΔH is the maximum height difference for drainage basin.

Since $h + \Delta h = \Delta H$, Equation (2-1) is also expressed as:

$$R_a = q \cdot \Delta h \cdot (1 - \Delta h/\Delta H) \tag{2-2}$$

Assuming that *a* is a point, using distance from *a* to the center of statistical figure or watershed as radius to calculate sampling area, the differential expression of relief amplitude is:

$$dh/da = q \cdot h \cdot (1 - h/\Delta H) \tag{2-3}$$

Therefore, relief height is:

$$h = \int (dh/da) = \Delta H/(1 + \exp(q' - q \cdot a)) \tag{2-4}$$

Equation (2-4) shows that the curve of relief height change with area is $Logistic$. q' is morphological parameter of geomorphic system which is equal to the ratio of the elevation of erosion basis and watershed elevation:

$$q' = H^*/H_0 \tag{2-5}$$

In order to explore the relief height change rule with area, this paper takes derivation of Equation (2-4) :

$$h' = q \cdot \Delta H \cdot \frac{e^{(q'-qa)}}{(1 + e^{(q'-qa)})^2} \tag{2-6}$$

where $h' > 0$, so relief height is a decreasing function.

Assuming $f = e^{(q'-qa)}$, Equation (2-6) becomes:

$$h' = q \cdot \Delta H \cdot \frac{f}{(1 + f)^2} \tag{2-7}$$

By taking derivation, Equation (2-7) becomes:

$$h'' = q \cdot \Delta H \cdot \frac{(1 - f^2) f'}{(1 + f)^4} \tag{2-8}$$

From Equation (2-8), numerator $(1-f^2)f'$ determines h'' is plus or minus. If $f=e^{(q'-qa)}$, it is defined as below:

$$h''\cdot(1-f^2)f'=q\cdot e^{(q'-qa)}\cdot(e^{2(q'-qa)}-1) \tag{2-9}$$

which is,

$$\begin{cases} q'>qa \quad ,h''>0 \quad h'\text{increasing function} \\ q'=qa \quad ,h''=0 \\ q'<qa \quad ,h''<0 \quad h'\text{decreasing function} \end{cases} \tag{2-10}$$

q' determines the changing speed of relief height on the space. When $aq<q'$, h increases quickly with increasing area. When $aq>q'$, h increases slowly with increasing area. When $a=q'/q$, $h=\Delta H/2$, which is the turning point for *Logistic* curve.

2.2. Simulation Equation

Equation (2-4) describes the the changing curve of relief amplitude increase. Its shape is an opposite *S*-type. When the area $a=q'/q$, increasing speed of h changes from being upward to downward, which is defined as the "inflection point" in geomorphology by Tu Han-ming [1]. The area a^* on this point is the area of statistical unit. Therefore, if we can get q' and q from Equation (2-4), a^* also can be obtained.

1) transform *Logistic* to linear equation

Taking $y'=\frac{1}{h}$, $A'=\frac{1}{\Delta H}$, $B'=\frac{e^{q'}}{\Delta H}$, $b'=-q$, $x=a$

Equation(2-4)becomes:

$$y'=A'+B'e^{b'x} \tag{2-11}$$

Taking $y=\ln(y'-A')$, $A=\ln B'$, $B=b'$

Equation(2-7)becomes:

$$y=A+Bx \tag{2-12}$$

2) get the parameters in *Logistic* equation with SPSS based on the principle of LSM.

$$B'=\frac{e^{q'}}{\Delta H}\text{ ,}\quad q'\ln e=\ln B'+\ln\Delta H\text{ ,}\quad \ln B'=A$$

$$q'=A+\ln\Delta H$$

$$\text{Thus, } a^*=\frac{q'}{q}=\frac{A+\ln\Delta H}{-B} \tag{2-13}$$

2.3. Applicability of Simulation Equation

From above definition, the method of finding out "inflection point" is that we get a^* by q' and q depended on the change rule of $\ln(\frac{1}{h}-\frac{1}{\Delta H})$ with unit area a. However, this method has to use the maximum elevation difference that is not applicable for present study. Simulation equation is just applied on that scholars use the highest frequency unit area as the optimum statistical unit based on the change rule of a single point in different unit area.

3. Data and Research Method

3.1. Data Sources and Processing

SRTM-DEM data used in this paper are measured by National Aeronautics and Space Administration (NASA) and National Imagery and Mapping Agency (NIMA). Its resolution is 90 meters. There are 65 picture cover nationwide original DEM data, which all adopt GCS_WGS_1984 as geographic coordinates. It is necessary to preprocess SRTM-DEM before analyzing and drawing, including data format conversion, geographic coordinates transformation, projection transformation and data registration, which transform binary data to ArcGIS raster data. Considering characteristics of the study area, this research uses Albers projection to output 100m*100m pixel and extracts relief amplitudes under different statistical units by ArcGIS.

3.2. Extraction of Relief Amplitude

There are four types analysis window of neighborhood analysis method: rectangular, circular, loop and sector. This paper uses rectangular window. The size of window is n×n pixel(n=2,3,4,...,22,23,24,25). Based on the SRTM-DEM data after pre-processing, using n×n pixel rectangular as operator model with zonal statistics of ArcMap, we can calculate the maximum(max) and the minimum (min) of n×n window, then using raster calculator to calculate the elevation difference between the maximum and the minimum which is the relief amplitude of n×n window.

4. Extraction of Relief Amplitude in China under Different Statistical Units

Based on SRTM-DEM, relief amplitude in China under different statistical units are extracted by neighborhood analysis in ArcMap. In order to calculate the optimum statistical unit, this paper summarizes the relation between n×n statistical units and relief amplitude in table 1.

Table 1: Relation between Statistical Units and Relief Amplitude

Unit size	Area($10^4 m^2$)	Relief amplitude(m)			
		Max	Ave	Number of highest frequency	Highest frequency
2×2	4	3922	25.24	90711109	2
3×3	9	3946	47.12	60079524	3
4×4	16	3946	66.44	46788895	4
5×5	25	4025	83.86	37993738	5
6×6	36	4025	99.8	33405211	5
7×7	49	4025	114.56	29236779	5
8×8	64	4025	128.31	26873752	6
9×9	81	4025	141.22	25020049	6
10×10	100	4025	153.4	23224983	6
11×11	121	4025	164.93	21542236	7
12×12	144	4025	175.9	20726933	7
13×13	169	4025	186.37	19893955	7
14×14	196	4025	196.39	19047202	7
15×15	225	4025	206	18180230	7
16×16	256	4025	215.25	17294710	7
17×17	289	4025	224.16	16683849	8
18×18	324	4025	232.76	16234801	8
19×19	361	4025	241.07	15773672	8
20×20	400	4025	249.13	15303703	8
21×21	441	4025	256.95	14833942	8
22×22	484	4025	264.54	14364943	8
23×23	529	4025	271.92	13893813	8
24×24	576	4025	279.11	13541187	9
25×25	625	4025	286.11	13277116	9

5. The Optimum Statistical Unit of Relief Amplitude in China

5.1. Inflection Point

Inflection point in physiognomy is the optimum statistical unit point. Liu Zhen-dong *et al* [3] mentioned that elevation difference is proportional to statistics unit area. The value of elevation difference grows rapidly at beginning, then becomes slowly. When area of model contains the whole, the growth is not obvious. And we call this area as individual optimum statistical unit. Liu Zhen-dong first identified the qualitative standard of inflection point in physiognomy in 1989. In 1990, Tu Han-ming *et al* [4] proposed to determine the optimum statistical unit with artificial drawing method, the maximum altitude difference method and fuzzy mathematics method. In 1991, Tu Han-ming established mathematical model and simulation equation of relief amplitude, and defined the mathematical meaning of inflection point in physiognomy [1].

In order to discuss the first question mentioned in Introduction, this paper according to equation derivation and contrastive analysis points out that inflection point defined by Liu Zhen-dong is the point tending towards stability, but inflection point defined by mathematical equation is the point when second derivative is zero. Generally, the optimum statistical unit area extracted from "stable point" is bigger than "inflection point in mathematics".

5.2. Research Idea

The main research idea of relief amplitude can be described as follows:

1) from point to area: research the change rule of a single point in different unit area, then select the highest frequency sample point as the optimum statistical unit.
2) directly from area: calculate the relief amplitude of all grid cells (sample points) in a unit area, then research the change rule.

The first idea can grasp the overall information more accurately, but the computation is complex and inefficient. The second idea is efficient because of usage of GIS, however, the accuracy of this idea does not be proofed yet. Therefore, the author thinks that it is necessary to study the difference and relativity between these two ideas.

5.3. Selection of Dependent Variable and Fitting Model

There are two disadvantages of existing method for determining the optimum statistic unit:

1) selection of dependent variable: there is no reference analyzes the applicability of maximum relief amplitude and average relief amplitude.
2) selection of fitting model: according to existing researches the change rule of relief amplitude conforms to Logistic model, actually they are not fitting. This paper thinks the idea whether choose maximum relief amplitude (or average relief amplitude) as dependent variable needs to be discussed further.

This paper aims at these problems to make regression analysis by SPSS, the fitting curves are shown in figure 1 to figure 4:

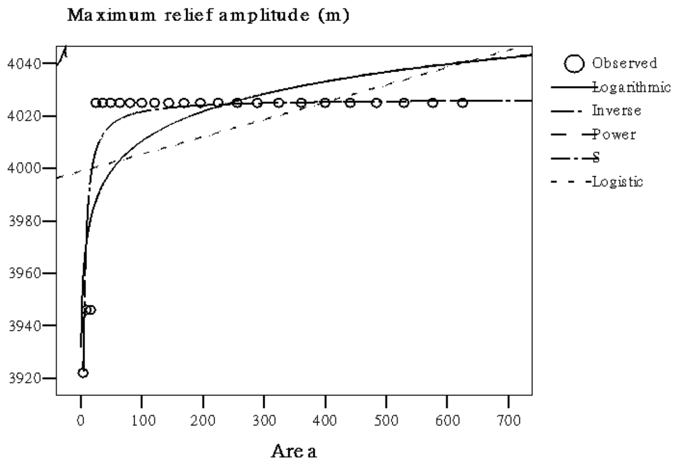

Fig. 1: Fitting curve of the relationship between unit area and maximum relief amplitude

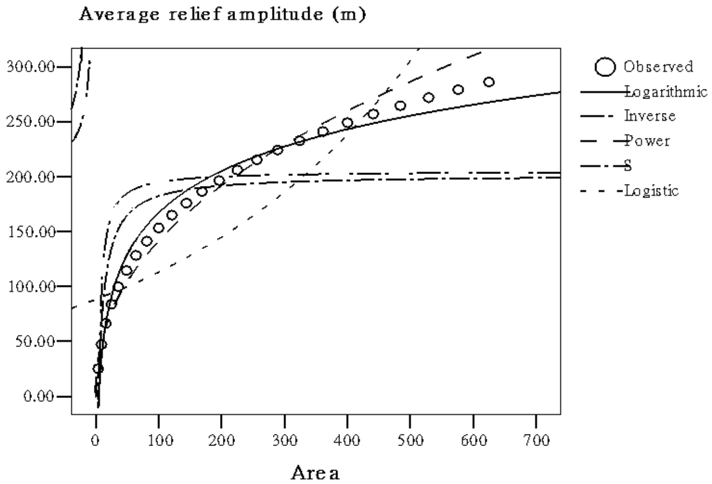

Fig. 2: Fitting curve of the relationship between unit area and average relief amplitude

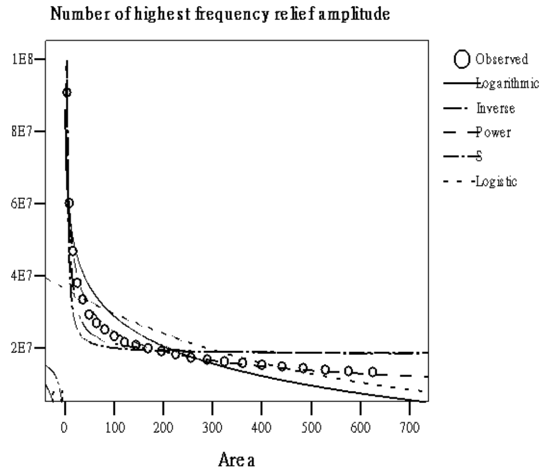

Fig. 3: Fitting curve of the relationship between unit area and highest frequency of relief amplitude.

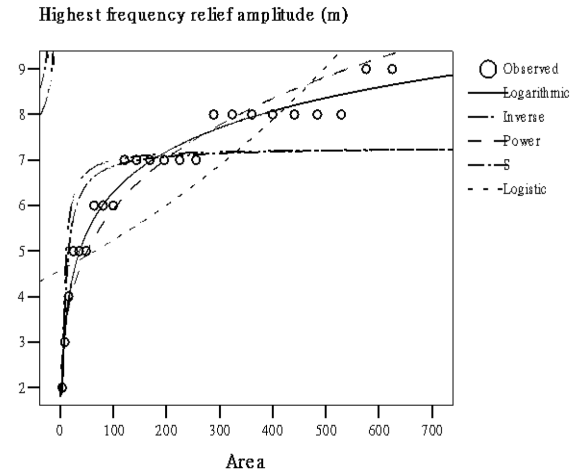

Fig. 4: Fitting curve of the relationship between unit area and relief amplitude of highest frequency.

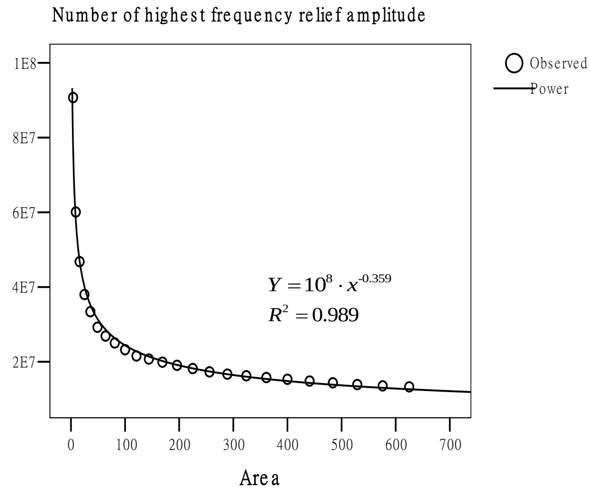

Fig. 5: Power model—highest frequency of relief amplitude

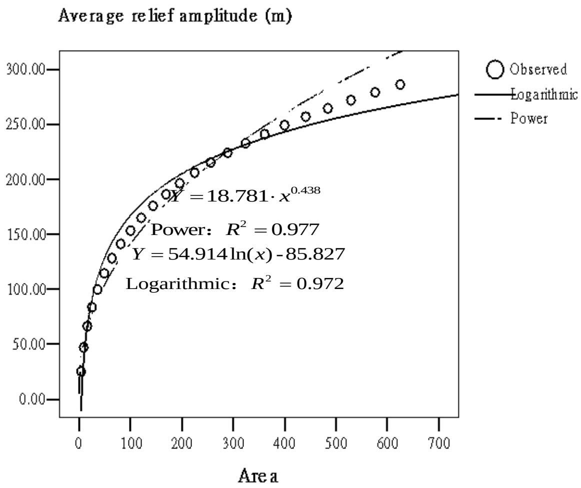

Fig. 6: Power and logarithmic model—average relief amplitude

$$Y = 1.289\ln(x) + 0.354$$
$$R^2 = 0.972$$

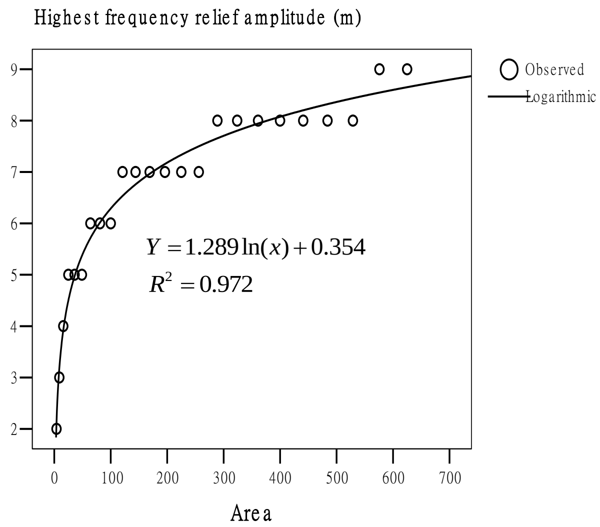

Fig. 7: Logarithmic model—relief amplitude of highest frequency

Table 2: Comparison of the Fitting Results (Determination Coefficient R2) between Different Models and Variable Curves

Model / Variable	Logarithmic	Inverse	Power	S	Logistic
Maximum relief amplitude	0.587	0.796	0.587	0.798	0.186
Average relief amplitude	0.972	0.494	**0.977**	0.779	0.624
Highest frequency of relief amplitude	0.855	0.937	**0.989**	0.735	0.666
Highest frequency relief amplitude	**0.972**	0.651	0.930	0.833	0.554

Comparison of the fitting results (determination coefficient R^2) between different models and variable curves is summarized in table 2. From table 2:

1) the imitative effect of “power model” is the best.
2) the maximum relief amplitude as dependent variable is not suitable for the optimum statistical unit research.
3) we would better use the highest frequency relief amplitude and the average relief amplitude as dependent variables when researching on the optimum statistical unit.
4) when choosing average relief amplitude as dependent variable, we had better use “ logarithmic model ” and “ power model ”.
5) the change rule of relief amplitude parameters does not conform to Logistic model, which means variables obtained by this idea are different from references, thus, methods mentioned in references[1], such as artificial drawing method, the maximum altitude difference method and fuzzy mathematics method, whether apply to inflection point search need to be discussed .

The relations between different fitting model and each dependent variable listed in figure 5 to figure 7, respectively, are described above.

Based on above data, for nationwide SRTM-DEM data, fitting equation which states the relation between “power model” and highest frequency of relief amplitude is $Y = 10^8 \cdot x^{-0.359}$. For average relief amplitude, the fitting equations of “power model” and “logarithmic model” are described as $Y = 18.781 \cdot x^{0.438}$ and $Y = 54.914\ln(x) - 85.827$, respectively. Fitting equation which states the relation between “logarithmic model” and the highest frequency relief amplitude is $Y = 1.289\ln(x) + 0.354$.

According to figure 5-7, the overall rule of fitting curve is that the value of the average relief amplitude and the highest frequency increases with increasing area, but the increasing rate is slowing down then becomes stable. The value of highest frequency of relief amplitude reduces with increasing area, and the decreasing rate is also slowing down then becomes stable. In addition, there are a point that becomes stable for three fitting curve described above, respectively. This point is the optimum statistical unit point, and the area in this point is the nationwide optimum statistical area.

6. Conclusion and Discussion

Based on ArcGIS, this paper analyzes mathematic definition, research idea, and selection of dependent variable and fitting model for China relief amplitude optimum statistical unit with SRTM-DEM data. The main conclusions are summarized as follows:

1) Tu Han-Ming [1] points out that the "inflection point" defined by mathematical equation is the same point in mathematics. When use artificial drawing method to determine the optimum statistical unit, we need to find out the point which becomes stable.
2) When we use SRTM-DEM data to study extraction of relief amplitude, "power model" is the best, and second best is average relief amplitude. Maximum relief amplitude is not suitable to be dependent variable.
3) The change rule of relief amplitude parameter extracted from second research idea does not conform to Logistic model. The relief amplitude variable from this idea is different from the math concepts in references [1].

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8. References

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