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OFDM System Channel Estimation Based on Compressive Sensing Technology

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Abstract. Compressive Sensing is applicable to the sparse signals or sampling signals, and can compress the signal data properly in course of sampling, therefore, it can carry out sampling at a rate much lower than that specified as per Nyquist Sampling Theorem, and reconstruct the original signals accurately. This paper mainly studies the channel estimation algorithm of OFDM system, including the traditional channel estimation algorithm and that based on compressive sensing theory, and makes a relevant analysis on their respective theories, thoughts and characteristics. Furthermore, an experiment simulation is conducted here to compare their performance in an all-round manner, and the conclusion indicates that the signal acquisition technology based on compressive sensing technology performs better. The compressive sensing can effectively solve the problems encountered by the traditional signal sampling and coding technology in the aspect of processing speed, memory space and anti-interference function, showing a promising application background.

Keywords: compressive sensing, signal sampling, signal reconstruction, channel estimation.

1. Introduction

With the rapid development of digital multimedia and communication, people have sought to reduce the burden of data acquisition, storage and transmission. Conventionally, Shannon's celebrated theorem, which indicates that the sample rate must be at least twice the maximum frequency present in the signal [1], underlies nearly all data acquisition protocol in real-world application. Compressive Sensing, which applies to the sparse signals or sampling signals [2] and aims to recover signals from incomplete linear measurements [3], is a relative new signal acquisition method internationally proposed in recent years [4], and can compress the signal data properly in course of sampling. Therefore, it can carry out sampling at a rate much lower than that specified as per Nyquist Sampling Theorem, and reconstruct the original signals accurately. The compressive sensing can effectively solve the problems encountered by the traditional signal sampling and coding technology in the aspect of processing speed, memory space and anti-interference function, showing a broad application background.

In the modern mobile communication system, in fact, the delay spread phenomenon is the main factor that contributes to selective fading of the frequency in the wireless channels. Furthermore, the common Doppler effects also causes this fading in the time domain. Therefore, these two fading would make the phase and amplitude of the signals undergo severe noises when passing the wireless channel, and leads to deterioration of performance in the whole communication system. By transforming the high-speed serial signal flow into low-speed parallel signal flow, the OFDM system can decrease the data transmission speed while increasing the duration of subcarrier data symbol, therefore reducing the interference among symbols and enhancing the bandwidth efficiency. Great progress and achievement have been made in the signal estimation algorithm of OFDM system, and the pilot method is just an excellent algorithm.

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In this paper, we propose a novel channel estimation algorithm, which utilizes the compressive sensing theory in estimating OFDM communication channels and achieves robust performance. Numerical experiments show that we can use the sparse values of channel that is known in advance to go beyond the restriction of Nyquist Sampling Theorem to reduce the number of pilot insertion, therefore, making the channel estimation more close to the actual value.

2. Theoretical Basis of Compressive Sensing

The compressive sensing technology has broken through the bottleneck of Shannon's sampling theorem, made the observation data required to reconstruct the original signals much less than that required by the traditional sampling method, and enabled the acquisition of high-resolution signals. The compressive sensing mainly focuses on three core issues: sparse expression of signal, sparse observation of signal and reconstruction of signal [5].

For a certain signal, we usually express it roughly as a linear combination of some linearly independent basis. Assume that a discrete signal with finite length $x \in R^{N\times 1}$, $\Psi = \{\psi_1, \psi_2, ,,, \psi_N\}$ is the orthogonal basis or dictionary of a certain $N \times N$, and the signal *x* under basis Ψ may be expressed as:

$$x = \sum_{i=1}^{n} \psi_i s_i = \Psi S \tag{1}$$

Wherein, $s_k = \langle x, \psi_i \rangle$, and $S = R^{N\times 1}$

Assume that a certain discrete signal $x \in R^{N\times 1}$ is $k-$sparse, namely, this signal contains at most k non-zero quantities; we can express it as $\|x\|_0 \le k$, wherein $\|\bullet\|_0$ refers to the number of non-zero element; we adopt the following equation to describe the $k-$sparse signal:

$$\Gamma_k = \left\{ x : \|x\|_0 \le k \right\} \tag{2}$$

A majority of signals may be expressed as the *k*- sparse signal under a certain basis.

The sparse observation of signal inevitably involves the design of measurement matrix [6], which plays a crucial role. The design of measurement matrix mainly follows the Uniform Uncertainty Principle and Restricted Isometry Property, and its arithmetic expression is as follows:

$$1 - \delta_k \le \frac{\|\Phi S\|_2^2}{\|S\|_2^2} \le 1 + \delta_k \tag{3}$$

The signal reconstruction serves as the most crucial part of compressive sensing [7]. It means reconstructing the N-dimension ($M \ll N$) k sparse signal x on the basis of vector quantity Y of M-dimension measurements. The norm is defined as follows:

$$l_p = \sqrt[p]{\sum_{i=1}^{n} |l_i|^p}, p \ge 0 \tag{4}$$

As for the mathematical solution, many existing mathematical conclusions can prove that the same results may be obtained at nearly full probability by seeking the norm minimization l_1. In other words, by solving the followings:

$$\hat{x} = \arg\min \|s\|_1 \text{ s. t., } \Phi X = \Phi \Psi S = Y \tag{5}$$

We can reconstruct the original signal accurately [8].

3. Sparse Channel Estimation of OFDM System

3.1. OFDM system

The functional block diagram of OFDM system is shown as follows:

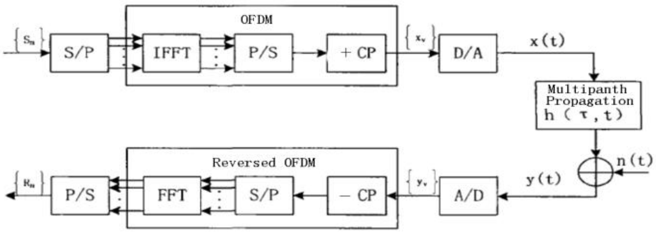

Fig. 1. Functional Block Diagram of OFDM.

The expression of received signal [9]:

$$y(t)=\sum_{j=1}^{p}\beta_j\sum s[n]e^{j2\pi\left(\frac{n}{t}+f_c\right)\left(t-\frac{j}{W}\right)}+z(t) \tag{6}$$

By conducting *M* sampling for the received signals, we get:

$$y[m]=\sum_{j=1}^{p-1}\beta_j x_j[m]+z[m] \tag{7}$$

Wherein, $x_j[m]$ refers to the discrete sampling signals that are sent out, and it may be expressed as:

$$x_j[m]=\sum s[n]e^{j2\pi\left(\frac{n}{T}+f_c\right)\left(m-\frac{j}{W}\right)}+z(m) \tag{8}$$

The equation (7) may be transformed into the more intuitive matrix expression:

$$\begin{bmatrix} y(1) \\ y(2) \\ .... \\ y(m) \end{bmatrix}=\begin{pmatrix} x_0(1) & \cdots & x_{p-1}(1) \\ \vdots & \ddots & \vdots \\ x_0(M) & \cdots & x_{p-1}(M) \end{pmatrix}\begin{bmatrix} \beta_0 \\ \beta 1 \\ ... \\ \beta_{p-1} \end{bmatrix}+\begin{bmatrix} z(1) \\ z(2) \\ .... \\ z(M) \end{bmatrix} \tag{9}$$

The matrix expression may be simplified as:

$$Y=XH+Z \tag{10}$$

Wherein, *Y* refers to the received signal, *X* the sent signal, *Z* the Gaussian white noise, *H* the unknown matrix. Assume that the channel is sparse, that is, *p* discrete quantities only have p_1 non-zero coefficients, and $p_1 \ll p$, then we can adopt the compressive sensing method to carry out the channel estimation [10].

3.2. Sparse channel estimation

For many communication environments, such as the multiple line system of broadband, hydro-acoustic communication system and deep space communication, their channels all have sparse characteristics. In equation (10), we conduct the Discrete Fourier Transform conversion at point *M* of both sides at the same time, and then convert the time-domain information into frequency-domain information for analysis and calculation. The convolution of time domain equals that of frequency domain, and the different OFDM signals do not interfere with each other. Assume that the *k*st subcarrier of the *i*st OFDM signal sends the frequency-domain signal $x(k)$ and receives the signal $y(k)$ [11]:

$$y(k)=H(k)x(k)+z(k) \tag{11}$$

Wherein $H(k)$ refers to the frequency response form of channel, which can be expressed as:

$$H(k)=\left[\omega_1,\ \ \omega_2,\ \ ...\ \ \omega_Q\right]B\left[\varphi_1,\ \ \varphi_2\ \ ...\ \ \varphi_M\right]^T \tag{12}$$

Wherein *B* refers to the channel amplitude matrix, which can be expressed as:

$$B=\begin{bmatrix} \eta_{0,0} & \eta_{1,0} & ... & \eta_{p-1,0} & 0 & ... & 0 \\ \eta_{0,1} & \eta_{1,1} & ... & \eta_{p-1,1} & 0 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... \\ \eta_{0,Q-1} & \eta_{1,Q-1} & ... & \eta_{p-1,Q-1} & 0 & ... & 0 \end{bmatrix} \tag{13}$$

In this way, the sparsity expression of channel just changes into that of channel amplitude. In the actual channel transmission system, the signal energy is distributed unevenly, and often concentrates on the finite number of multi-path components.

The OFDM system channel [12] has different pilot types and different insertion modes, which exerts a direct influence on the accuracy of channel estimation; the pilot inserted into the OFDM system mainly includes the following types:

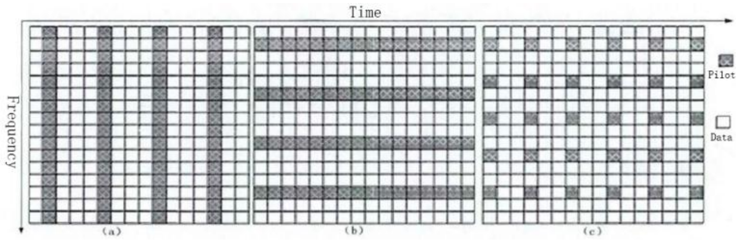

Fig. 2: Pilot Scheme: (a) block-type pilot; (b) comb-type pilot; (c) interval-type pilot.

Actually, the pilot insertion process is also a sampling process. According to the traditional sampling principle, the pilot insertion interval must comply with the Nyquist Sampling Theorem so as to get an accurate value. Assume that the minimum sampling or insertion interval in the direction of time and frequency domain is N_f and N_t respectively, if we don't want to impair the frequency-domain and time-domain signal, we can get:

$$N_f \le \frac{1}{\tau_{\max} \Delta F_c} \tag{14}$$

$$N_t \le \frac{1}{2 f_d T} \tag{15}$$

Wherein, ΔF_c refers to the sub-carrier interval, and f_d the Doppler spread. In the practical application, both N_f and N_t usually need to be integer.

As is known according the OFDM system principle [13], the pilot signal was added at the signal transmitting end. If we denote the pilot selection symbol matrix as R, the whole pilot process may be expressed as follows:

$$Y_p = RY = R \bullet diag\left(X\right)Fh + R \bullet Fn = diag\left(X_p\right)Fh + R \bullet Fn \tag{16}$$

If we replace $R \bullet diag\left(X\right)F$ with matrix Φ, and $n^{'}$ with $R \bullet Fn$, the equation (16) will be converted into:

$$Y_p = \Phi h + n^{'} \tag{17}$$

Obviously, we may regard Y_p and Φ as the observed value and sensing matrix in the compressive sensing process, wherein, h refers to the sparse solution that we want. The compressive sensing method may be applied to get the estimation solution for channel h [14].

4. Experiment Simulation

In order to find out the difference in channel estimation performance between the compressive sensing-based on orthogonal matching pursuit (OMP) method [15], the traditional least-square estimation (LS) and the least mean-square estimate (MMSE), this paper made the following simulation: assume that the OFDM sparse channel remains unchanged within one symbol; the sub-carrier number of OFDM is 256; the QPSK modulation is adopted for the sent series; the OFDM channel uses the multipath sparse channel featuring multi-frequency fading; the selected path delay will be the integral multiple of sampling period in order to facilitate the research. The channel length is set as 30; the sparsity K is 5; the pilot number chosen for the simulation herein is 4-6 times of the sparsity, that is, 20 and 30.

When selecting the pilot number P=20, the normalized mean squared error (MSE) of such 3 estimation methods as LS, MMSE and compressive sensing OMP are compared as follows:

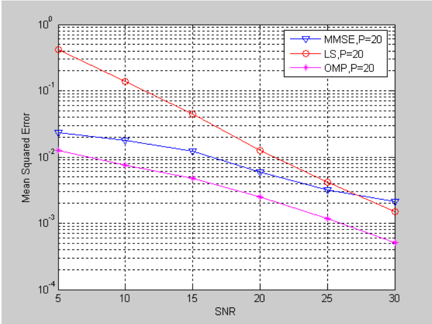

Fig. 3: MSE Comparison between LS, MMSE and OMP Algorithm (P=20).

When selecting the pilot number P=30, the MSE of such 3 estimation methods as LS, MMSE and compressive sensing OMP are compared as follows:

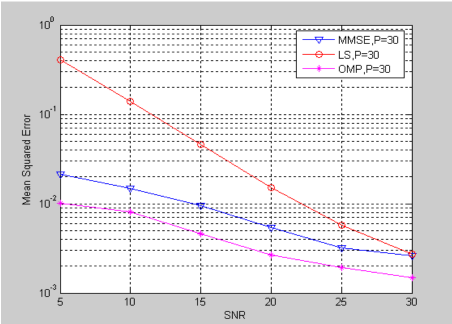

Fig. 4: MSE Comparison between LS, MMSE and OMP Algorithm (P=30).

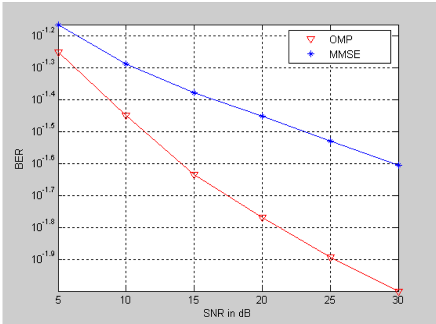

Fig. 5: BER Comparison between MMSE and OMP Algorithm.

The bit error rate (BER) of such two estimation methods as MMSE and compressive sensing OMP are compared as follows:

By comparing Fig. 3 with Fig. 4, we can see that the MSE of LS, MMSE and compressive sensing OMP algorithm will all decrease in various degrees with the increase in pilot number [16]. When the Signal Noise Ratio (SNR) of OFDM system is less than 5dB, we don't see much difference between the MMSE algorithm and the compressive sensing OMP algorithm, that is, the latter does not perform significantly better at this time. However, with the increase in SNR of system, the compressive sensing technology will show more obvious advantages. As can be seen from the simulation diagram, in order to achieve the same performance, the channel estimation algorithm based on compressive sensing needs much less pilot numbers than the traditional channel estimation algorithm, and the saved pilot number may be used to transmit data; that means, the compressive sensing technology can be used to further enhance the spectrum efficiency, therefore constituting a better channel estimation algorithm. According to the BER comparison shown in Fig. 5, the compressive sensing OMP algorithm has a lower channel BER estimation result. As the SNR of system increases gradually, the OMP algorithm will show a significantly better performance, because the low SNR (namely a higher Gaussian white noise in the channel) means that the compressive sensing technology is inaccurate at the beginning of sampling, which will lead to the failure of OMP algorithm at the time of reconstruction.

5. Conclusions

By studying the application of compressive sensing theory in the OFDM channel estimation, this paper mainly analyzes the OFDM channel model and the compressive sensing-based OFDM channel model, and introduces their theories, mathematical basis and mathematical derivations in a detailed and step-by-step manner. A simulation experiment is done for the traditional channel estimation algorithm (LS and MMSE) and the channel estimation based on compressive sensing theory in order to find out the latter's advantages in channel estimation by analyzing their performance index (MSE and BER).

The simulation results show that the compressive sensing-based OFDM channel estimation algorithm uses less pilot number, but gives a better estimation results for channel H. When the result of channel estimation is applied in the system equilibrium of OFDM communication system, we can see that the compressive sensing-based method leads to a lower BER, enhances the bandwidth efficiency and increases the throughput of OFDM system. According to the channel estimation method newly proposed in this chapter, we fully believe that the compressive sensing technology will be more extensively applied in the field of signal estimation in the future.

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7. References

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