

# Implementation of an Efficient Multi-Signature Scheme for Copyright Protection on Ethereum

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**Abstract.** Blockchain has shown great potential in various fields due to its technical advantages such as peer-to-peer, timestamp, consensus algorithm and encryption. In blockchain, the protection method of transaction data or copyright is crucial and cryptographic digital signature technology has been applied as one of the copyright protection methods. The multi-signature scheme provides higher security than single-signature schemes, enhances the transparency of transactions and contracts, and is widely used in distributed systems utilizing distributed ledger technology in blockchain. Multi-signature requires multiple parties to cooperate in order to produce a valid signature, reducing the risk of exposing the entire system to a single point of failure when compared to single-signature schemes. This is an important role in transactions or contracts that require consensus among multiple parties, where each party can sign to implement the agreement, increasing transparency and preventing disputes. However, the cryptographic digital signature is resource consuming and inefficient because the verification of the signature consumes lots of computational resources and excessive number of communications. Therefore, we have proposed an efficient multi-signature scheme based on Schnorr for copyright protection on Ethereum.

**Keywords:** Schnorr, multi-signature, Ethereum, copyright protection, digital signature

## 1. Introduction

Since the Ethereum blockchain platform was published, blockchain is studied rapidly to implement and functions that can be applied [1]. In blockchain, the protection method of transaction data or copyright is crucial and cryptographic digital signature technology has been applied as one of the protection methods. Digital signature technology can guarantee the authenticity, integrity, and non-repudiation of data in electronic commerce and electronic government affairs. The signature technique is not affected by the number of signatures, the final result is only one digital signature in length. Signature can mitigate the above deficiencies or bottlenecks faced by the system based on blockchain [2].

A transaction involving multi-parties needs signatures to take effect, and it will consume a large amount of storage on the blockchain. Schnorr signatures feature a rich algebraic structure suitable for the construction of multiparty protocols. Also, it can be further utilized to construct other cryptographic primitives like multi-signatures, threshold signatures, blind signatures, or adaptor signatures, all which have interesting applications especially to lately arising blockchain technologies. Individual cosigners do not reveal their private information during this process, multi-signature schemes allow for the decentralization of the signing process by requiring multiple parties to cooperate in order to produce a valid signature. A trivial multi-signature scheme can be constructed using an arbitrary digital signature scheme in the following manner. The signers generate their key pairs, individually sign the message, and concatenate their signatures. The verification then proceeds by obtaining all the public keys of the signers and verifying concatenated signatures individually [3]. The length of the multi-signature was reduced to a constant. A single aggregated key can be used to verify the multi-

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signature. And the individual signatures are bound to the resulting multi-signature; therefore, they cannot be removed. However, the cryptographic digital signature is resource consuming and inefficient because the verification of the signature consumes lots of computational resources and excessive number of communications. Therefore, we have proposed a secure and efficient multi-signature scheme based on Schnorr in this paper.

The rest of this paper is organized as follows. We present the digital cryptographic single and multi-signature algorithms based on Schnorr in section 2. In section 3, we describe the experimental results show that the proposed our multi-signature scheme based on Schorr can effectively improve the performance for copyright protection and reduce communication overhead on Ethereum. Finally, section 4 concludes this paper.

## 2. Schnorr Signature

Schnorr signature has a characteristic of fast sign speed and short signature length. It usually uses the P-256 curve released by NIST (national Institute of Standards and Technology) in the United States, and uses sec256r1 curve by SEC. It has been proven strongly unforgeable against adaptively chosen-message attacks in the random oracle model (ROM) [4] by Pointcheval and Stern [5]. We have represented a message,  $m$  and a private key,  $q$  as input values in our implementation of signing and verification of Schnorr scheme as in Fig.1 and Fig. 2. In a  $k$  value, we hash the private key and the message with the keccak256 hash function and obtain two public keys (x, y) coordinates by multiplying the generator point  $G$  in  $q$  by the elliptic curve scalar. Find the equation  $e = \text{hash}(kG \parallel Q \parallel M)$  required for Schnorr signature and make  $s = k - eq$  the equation. The public keys,  $s$ , and  $e$  signature values are outputs as return values. It has a message and , public key,  $Q$ , and  $s$ , and  $e$  as input values that are result values from the signature algorithm in verification algorithm of our scheme. If the public key has an infinite value or does not exist in the elliptic curve, return value is *False*. As long as the equation  $kG = sG + eQ$  is performed and the result of hash with the  $e$  signature value received as an input value is compared with the equation  $e = \text{hash}(kG \parallel Q \parallel M)$ . If it is wrong return *False*, else if right return *True*. Equation (1-3) performed the verification below.

$$sG + eQ = (k - eq)G + eqG \quad (1)$$

$$= kG - eQ + eQ \quad (2)$$

$$= kG = K \quad (3)$$

Schnorr Signing Algorithm
1: <b>function</b> SCHNORR-SIGN( $m, q$ )
2: $M \leftarrow \text{hash}(m)$
3: $k \leftarrow \text{int}(\text{hash}(\text{bytes32}(q) \parallel M))(\text{mod } n)$
4: $K \leftarrow kG$
5: $Q \leftarrow qG$
6: <b>if</b> $y_k \neq 1$ <b>then</b>
7: $k \leftarrow n - k$
8: <b>end if</b>
9: $e \leftarrow \text{int}(\text{hash}(\text{bytes32}(kG) \parallel Q \parallel M))$
10: $s \leftarrow (k - eq(\text{mod } n))(\text{mod } n)$
11: <b>return</b> ( $Q, \text{bytes32}(s), \text{bytes32}(e)$ )
12: <b>end function</b>

Fig. 1: Schnorr signing algorithm

Schnorr Verification Algorithm
1: <b>function</b> SCHNORR-VERIFY( $m, Q, s, e$ )
2: <b>if</b> $Q = \infty$ <b>or not curve</b> ( $Q$ ) <b>then</b>
3: <b>return</b> <i>False</i>
4: <b>end if</b>
5: $M \leftarrow \text{hash}(m)$
6: $kG \leftarrow sG + eQ$
7: <b>if</b> $e = \text{int}(\text{hash}(\text{bytes32}(kG) \parallel Q \parallel M))$
<b>then return</b> <i>True</i>
8: <b>else</b>
9: <b>return</b> <i>False</i>
10: <b>end if</b>
11: <b>end function</b>

Fig. 2 Schnorr verification algorithm

Due to the characteristic property of SMPC (Secure Multi-Party Computation), individual cosigners do not disclose their private information [6] and the signers generate their key pairs, individually sign the message, and concatenate their signatures. The verification proceeds by obtaining public keys of all the signers and verifying concatenated signatures individually [7]. Every single transaction that happened on the network will be signed and recorded in the blockchain. A transaction involving multi-parties needs multiple signatures to take effect, and it will consume a large amount of storage if everyone signs the transaction and stores the record on the blockchain. Every node on the blockchain keeps the same ledger, records need to be as compact as possible.

So, the multi-signature technique was applied to reduce bandwidth and storage volume in some services. The multi-signature technique based on Schnorr scheme is especially attractive for blockchain where communication and storage are more expensive than computation. It produces a multi-signature of constant length instead of linear in the number of signers. The advantage of having multi-signature of constant length, other than the apparent implications of smaller size, is faster multi-signature verification. The multi-signature verification time can become even constant if another requirement, described in the following paragraph, is satisfied.

Our proposed multi-signature signing scheme has described in pseudocode in Fig. 3. We have represented  $HASH\_CAL()$  for computing communication and  $HASH\_AGGR()$  for computing coefficients in key aggregation phase and  $HASH\_CAL()$  for used to compute a challenge in the signing procedure of hash function. A random private key  $Q_i$  is sampled from  $\mathbb{Q}_i$ , and the group generator  $G$  is multiplied by this private key to obtain  $Q_i = q_i G$ . The resulting key pair is  $(q_i, Q_i)$ . Each key is prior to that multiplied by a factor  $a_i$  which is dependent on the whole set of public keys of signers and also on the multiplied key itself. The factors  $a_i$  are computed as  $a_i = HASH\_AGGR(P_i, Q_i)$ , and the group key is then the sum  $Q = \sum_{i=1}^n a_i Q_i$ . Each signer  $i$  initiates the algorithm with a multi-set of public keys  $P = \{Q_1, Q_2, \dots, Q_n\}$  and its private key  $Q_j$ . The algorithm then computes  $a_i$  of the specific signer and the aggregated group key  $Q = MERGER(P)$ . The participants sample a nonce  $r_i$  from  $\mathbb{Q}_i$  and multiply the group generator by this number to obtain  $Q_i = r_i G$ . Once they have  $Q_i$ , each signer computes its commitment  $t_i = HASH\_CAL(Q_i)$ . The commitments are distributed to other cosigners. Upon receiving all commitments  $t_j$ , signers distribute theirs  $Q_i$  to other cosigners. The signers have to compute  $e = SCHNORR\_SIGN(m, q)$  and their partial signatures  $s_i = q_i + ea_i q_i \pmod{n}$ . The partial signatures are then distributed to other cosigners, and the participants compute  $s = \sum_{i=1}^n s_i \pmod{n}$ . The resulting multi-signature is the pair  $(Q, s)$ . As shown in Fig. 4, the verifier computes  $e = SCHNORR\_SIGN(m, q)$  and if the equation  $sG = Q + eQ$  holds, the signature is valid, otherwise, it is invalid.

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**Schnorr Multi-Signing Algorithm**

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1: function SCHNORR-MULTI-SIGN(m, P, q(i))
2:   a(i) = HASH-AGGR((P), Q(i))
3:   Q = MERGER(P)
4:   k(i) ← Q(q)
5:   Q(i) = r(i)G
6:   t(i) = HASH-CAL(Q(i))
7:   send t(i) to other cosigners
8:   send Q(i) to other cosigners
9:   if HASH-CAL(Q(i)) ≠ t(i)
10:    break
11:   for i = 1 to n
12:     Q = Q + Q(i)
13:   e = HASH_SIGN(m, Q)
14:   s(i) = r(i) + ea(i)q(i) (mod n)
15:   send s() to other cosigners
16:   for i = 1 to n
17:     s = s + s(i) (mod n)
18:   return(Q, bytes32(s))
19: end function

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Fig. 3: Multi-Signature signing algorithm

### 3. Simulation Results

Ethereum can access the main network, test network, and private network through the Go-Ethereum (Geth) client. Geth [8] is a client that allows access to the Ethereum blockchain network and is written in the Go language. We have used the so-called Ganache, which corresponds to the private network.

Solidity language [8] was designed by the Ethereum Solidity team, starting with Gavin Wood's proposal in August 2014, to implement Ethereum smart contract functionality operating on EVM(Ethereum Virtual Machine) in a static type of programming language. Solidity language is compiled into byte codes that can

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**Schnorr Multi-Verification Algorithm**

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1: function SCHNORR-MULTI-VERIFY(m, Q, s)
2:   e = HASH_SIGN(m, Q)
3:   if sG = Q ⊕ eQ
4:     return True
5:   else
6:     return False
7: end function

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Fig. 4: Multi-signature verification algorithm

operate on EVMs, and developers can implement applications by putting their own business logic into smart contracts through Solidity language. Items recorded in smart contracts cannot be denied and cannot be modified once deployed, so they must be carefully written. Remix IDE [9] is an integrated development environment site that enables integrated coding, compilation, debugging, and deployment of Solidity languages on a browser. It can be used in a web browser without a separate installation, has its own compiler ‘solc’, and can access Ethereum's network. As shown in Fig. 5, secret means a private key, and message can be viewed as the value to be signed. It can be seen that two public key values and  $s$  and  $e$  values, which are signatures required, appear as result values. The SCHNORR-VERIFY function is an algorithm that verifies  $s$ ,  $e$  signatures, public keys, and messages as input values in the Schnorr signature algorithm.

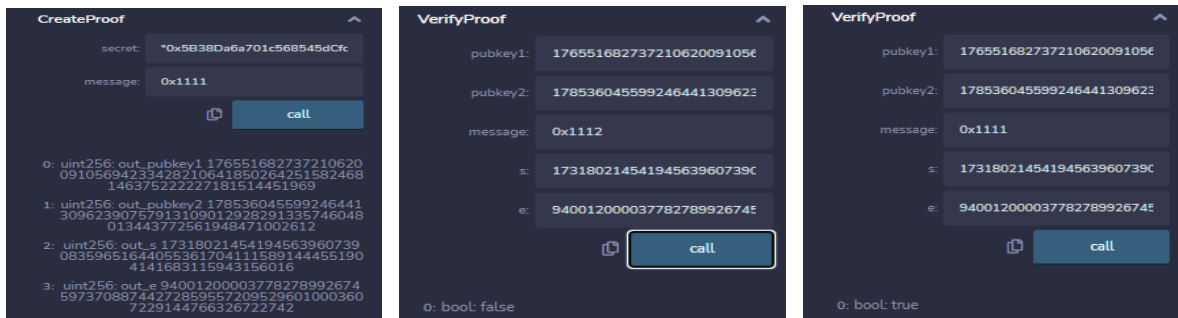


Fig. 5: Verification of Schnorr signature (false, true)

In the case that the number of signers is 5 and the random number has been generated, the comparison of the operation of single and multi-signature based on Schorr scheme have shown in Fig. 6. We found that multi-signature scheme is much more efficient in both signing and verification than single signature based on Schnorr scheme on the same situations. The signature length of 2048 bits will also significantly increase the overhead of system operation and storage. As shown in Fig. 7, we have compared the running time of the single and multi-signature based on Schorr scheme when the number of signers is five. The execution time of verification algorithm is not related to the number of signers but the length of the message, so it can be reduced the execution time by performing abstraction for large files, and our multi-signature scheme has shown high efficiency and it will be expected to be applied efficiently in copyright protection on Ethereum.

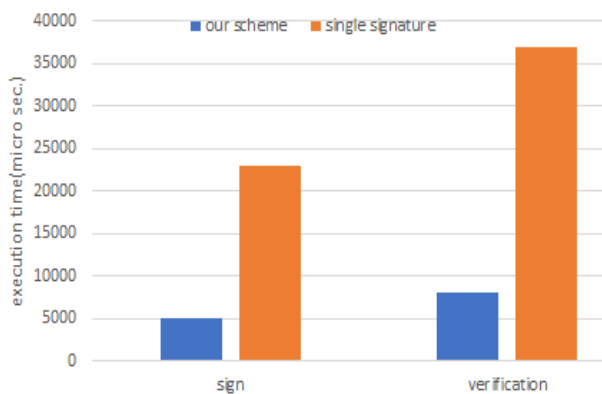


Fig. 6: execution time of signing and verification

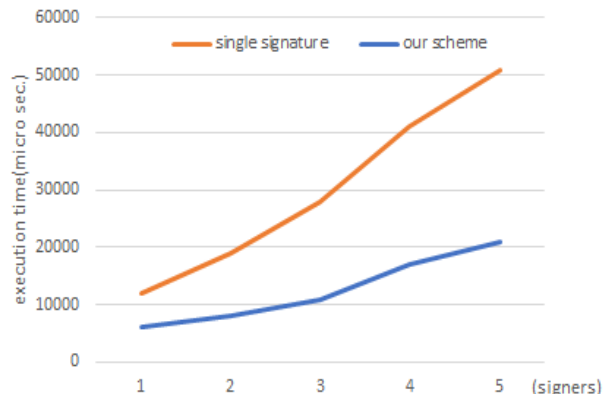


Fig. 7: execution time of multi-signers

#### 4. Conclusion

In this paper, we have proposed an efficient multi-signature scheme based on Schnorr for copyright protection on Ethereum. We describe the experimental results show that the proposed our multi-signature scheme based on Schorr can effectively improve the performance for copyright protection and reduce communication overhead on Ethereum. Also, we have compared the running time of the single and multi-signature based on Schorr scheme when the number of signers is five. We have obtained the result that multi-signature scheme is much more efficient in both signing and verification than single signature based on Schnorr

scheme on the same situations. Based on the simulation results in Section 3, our scheme is expected to be applied efficiently in copyright protection on Ethereum.

## 5. Acknowledgements

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