ISBN 978-981-14-4787-7
Proceedings of 2020 the 10th International Workshop on Computer Science and Engineering (WCSE 2020)
Yangon (Rangoon), Myanmar (Burma), February 26- February 28, 2020, pp. 84-89
doi: 10.18178/wcse.2020.02.015

Compressed Sensing Image De-noising Algorithm Based on L1-L2 Norm Regularization

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Abstract. In this paper, we propose a compressed sensing image de-noising algorithm based on L1-L2 norm regularization. After the image is decomposed by the total variation spectral framework, L1 norm regularization is performed on the texture image, and L2 norm regularization is performed on the contour image, then the alternating direction method of multipliers (ADMM) is used for solution. The results of numerical experiment show that the proposed algorithm obtains higher peak signal-to-noise ratio (PSNR) and structural similarity (SSIM) than the compared compressed sensing algorithm and the total variation algorithm, and can effectively maintain the contour information and texture information of the image when de-noising.

Keywords: compressed sensing, regularization, ADMM, total variation.

1. Introduction

Image noise is a problem in the field of image processing, and the noisy image will bring great difficulties to the feature extraction of the in the later stage[1].Therefore, the effective image de-noising has been the focus of attention. The purpose of image de-noising is to recover unknown original picture from the noisy image .In 1992, Osher and Fatemi proposed a classical total variation de-noising model [2]. However, the traditional total variation algorithm will misjudge the noise as the edge of the image, leading to staircase effect, which makes the quality of de-noised image unsatisfactory. On the other hand, the improved compressed sensing algorithm[3] based on the traditional total variation model can quickly reconstruct and de-noise the image, but the texture and structural features of the image are not taken into account.

In order to overcome this difficulty, based on the compressed sensing algorithm of reference [4], we propose a compressed sensing de-noising algorithm based on L1-L2 regularization. First, the noisy image is decomposed by the spectral framework to obtain a contour image with a small amount of noise and a texture image with a lot of noise. Since the contour image is mainly a smooth region, and is low-frequency information; while the texture image mainly contains details and noise, and is a high-frequency region [5], therefore, inspired by the reference[6], different weighting methods are used for the contour image and the texture image according to the structural features of the image. The L2 weighting method is adopted for the contour image with a small amount of noise, and the L1 weighting is adopted for the texture image with a large amount of noise, which can help to avoid the staircase effect in de-noising. Then, by combining the two parts after decomposition with the compressed sensing algorithm, a complex model with two regular terms is obtained. In order to solve this complex model, the alternating direction method of multipliers [7] (ADMM) similar to that in reference [8] is used for iterative solution. However, different from reference [8], in this paper, the Barziilai-Borwein method [9] in the gradient descent algorithm is directly adopted to solve the first sub-problem of the ADMM algorithm. Finally, the experiment results demonstrate that the proposed

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method is effective, and the objective PSNR and SSIM are superior to those of other algorithms of compressed sensing reconstruction.

2. Theoretical Foundation

The classical total variation regularization model [2] is as follows:

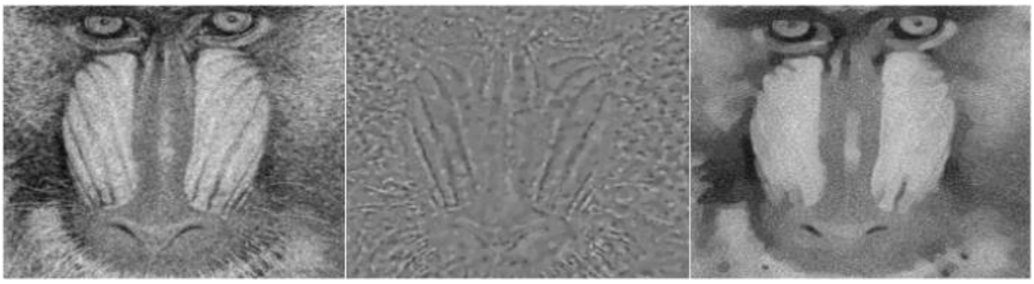
$$u = \arg \min_u \left(\frac{\kappa}{2}\int_\Omega |u - f|^2\, du + \int_\Omega |Du| du\right) \tag{1}$$

where u is the original image, f represents noisy image, D is the gradient operator of forward difference, Ω is the domain of the image, κ is the parameter. The first term is a fitting term to ensure that the de-noised image is close to the original image; the second term is a regular term, which is also known as the total variation of u, and can promote the smoothness of the de-noised image.

The high-dimensional original value u is recovered from the low-dimensional observed value $f = Au + b$, where $f \in R^M$, $u \in R^N$, $A \in R^{M*N}$ ($M \ll N$) is the observation matrix. This kind of reconstruction problem is essentially an ill-posed inverse problem, which can be solved by optimization algorithm. Therefore, the total variation model combined with compressed sensing is as follows:

$$u = \arg \min_u \left(\frac{\kappa}{2}\int_\Omega |Au - f|^2\, du + \int_\Omega |Du| du\right) \tag{2}$$

The total variation spectral framework, which was proposed by Gilboa, can decompose the original image into contour image and texture image [10, 11]. The image u can be decomposed into two parts: contour u_L and texture u_H.



(a) original image (b) contour image (c) texture image

Fig. 1: Images obtained by the decomposition of noisy image

3. Proposed Algorithm

The compressed sensing de-noising algorithm based on L1-L2 norm regularization is improved based on the compressed sensing algorithm proposed in reference [4], and it combines the advantages of weighted regularization and compressed sensing. In order to maintain the texture and edge information of the image while avoiding the staircase effect, the decomposed images are weighted and regularized:

$$\rho(u) = w_H \|Du_H\|_1 + w_L \|Du_L\|_2 \tag{3}$$

where $w_H = \eta/(1+\eta)$, $w_L = 1/(1+\eta)$, η is the absolute value of the image gradient mean.

The compressed sensing de-noising algorithm model based on L1-L2 norm regularization is:

$$(u_H, u_L) = \arg \min_{(u_H, u_L)} \left(w_H \|Du_H\|_1 + w_L \|Du_L\|_2 + \frac{\kappa}{2}\|Au - f\|_2^2\right) \tag{4}$$

The de-noising model (4) is a complex model with two regular terms. In order to facilitate the solution, ADMM is used to solve u_H and u_L respectively.

Fix u_L, the sub-problem of u_H can be obtained:

$$u_H = \arg \min_{u_H} \left(w_H \|Du_H\|_1 + \frac{\mu_H}{2}\|Au_H - f_H\|_2^2\right) \tag{5}$$

Fix u_H, the sub-problem of u_L can be obtained:

$$u_L = \arg \min_{u_L}(w_L \|Du_L\|_2 + \frac{\mu_L}{2}\|Au_L - f_L\|_2^2) \tag{6}$$

where $f_H = f - Au_L$, $f_L = f - Au_H$, μ_H and μ_L are the parameters. Obviously the problem after decomposition is easier to be solved than the original problem (4).

It is difficult to solve the sub-problem (5) directly. Therefore, the variable splitting [12] can be used to introduce the auxiliary variables u_1 and d_H. Set $u_1 = Au_H$ and $d_H = Du_H$. Then the sub-problem (5) can be converted to a constrained optimization problem.

$$\begin{aligned} u_H &= \arg \min_{u_H}(w_H \|Du_H\|_1 + \frac{\mu_H}{2}\|Au_H - f_1\|_2^2) \\ s.t.\ & u_1 = Au_H \quad d_H = Du_H \end{aligned} \tag{7}$$

The problem (7) can be converted to an unconstrained optimization problem using the Lagrange Multiplier Method, and its corresponding Lagrangian function is as follows:

$$\begin{aligned} L_1(u_H, d_H, u_1; \gamma_H, \delta_H; \mu) &= \|d_H\|_1 + \frac{\mu_H}{2w_H}\|u_1 - f_H\|_2^2 + \frac{\beta_{H1}}{2}\|Au_H - u_1\|_2^2 \\ &+ \frac{\beta_{H2}}{2}\|Du_H - d_H\|_2^2 - \gamma_H^T(Au_H - u_1) - \delta_H^T(Du_H - d_H) \end{aligned} \tag{8}$$

where β_{H1} and β_{H2} are penalty term coefficients, γ_H and δ_H are augmented Lagrangian coefficient matrix.

Since equation (8) contains three variables: u_H, d_H and u_1, it is complicated to solve. The ADMM can be used again to decompose it into three sub-problems with only one variable. Given u_H^K, d_H^K, u_1^k, γ_H^k, δ_H^k and μ_H^k ,we obtain

$$\begin{cases} u_H^{k+1} = \arg \min_{u_H} L_1(u_H, d_H^k, u_1^k; \gamma_H^k, \delta_H^k; \mu_H^k) \\ d_H^{k+1} = \arg \min_{d_H} L_1(u_H^{k+1}, d, u_1^k; \gamma_H^k, \delta_H^k; \mu_H^k) \\ u_1^{k+1} = \arg \min_{u_1} L_1(u_H^{k+1}, d_H^{k+1}, u; \gamma_H^k, \delta_H^k; \mu_H^{k+1}) \\ \gamma_H^{k+1} = \gamma_H^k - \beta_{H1}(u_1^{k+1} - Au_H^{k+1}) \\ \delta_H^{k+1} = \delta_H^k - \beta_{H2}(d_H^{k+1} - Du_H^{k+1}) \end{cases} \tag{9}$$

1) Fix d_H^K and u_1^k to solve u_H^{k+1}, u_H can be expressed as:

$$\begin{aligned} u_H = \arg \min_{u_H}(&\frac{\beta_{H1}}{2}\|Au_H - u_1\|_2^2 + \frac{\beta_{H2}}{2}\|Du_H - d_H\|_2^2 \\ &- \gamma_H^T(Au_H - u_1) - \delta_H^T(Du_H - d_H)) \end{aligned} \tag{10}$$

Gradient descent can be adopted to solve u_H, $u_H^{k+1} = u_H^k - \alpha_H^k g_H^k$, where g_H^k can be obtained by derivation, and α_H^k can be obtained by Barzilai-Borwein method [9].

$$\begin{aligned} g_H^k &= \beta_{H1}A^T(Au_H - u_1^k) + \beta_{H2}D^T(Du_H - d_H^k) - A^T\gamma_H^k - D^T\delta_H^k \\ \alpha_H^k &= \frac{v_H^{k\,T} v_H^k}{v_H^{k\,T} y_H^k} = \frac{(u_H^k - u_H^{k-1})^T(u_H^k - u_H^{k-1})}{(u_H^k - u_H^{k-1})^T[g_H^k(u_H^k) - g_H^k(u_H^{k-1})]} \end{aligned} \tag{11}$$

2)Fix u_H^{k+1} and u_1^k to solve d_H^{k+1}, d_H can be described as :

$$d_H = \arg \min_{d_H}(\|d_H\|_1 + (\beta_{H2}/2)\|Du_H - d_H + \delta_H^k/\beta_{H2}\|_2^2) \tag{12}$$

Equation (12) can be solved by the Shrinkage method [13]:

$$d_H^{k+1} = \max(\|Du_H^{k+1} + \delta_H^k/\beta_{H2}\|_1 - 1/\beta_{H2}, 0) * (Du_H^{k+1} + \delta_H^k/\beta_{H2}) / \|Du_H^{k+1} + \delta_H^k/\beta_{H2}\|_1 \tag{13}$$

3)Fix u_H^{k+1} and d_H^{k+1} to solve u_1^{k+1}, u_1 can be expressed as:

$$\min_{u_1}[(\mu_H / 2w_H)\|u_1 - f_H\|_2^2 + (\beta_{H1} / 2)\|u_1 - (Au_H + \gamma_H / \beta_{H1})\|_2^2] \tag{14}$$

Set the derivation of equation (14) equals to zero, then

$$u_1^{k+1} = \frac{\beta_{H1}(Au_H^{k+1} + \frac{\gamma_H^{k+1}}{\beta_{H1}}) + f_H \mu_H^{k+1} / w_H}{\beta_{H1} + \mu_H^{k+1} / w_H} \tag{15}$$

where the update of μ needs to meet the deviation criteria[14]. Set $\theta_H = \tau(\sigma_H^{k+1})^2 MN$, where σ_H^{k+1} is the noise standard deviation of f_H^k , MN is the total number of pixels in the image. If $\|u_1^{k+1} - f_H^k\| < c_H^{k+1}$, then μ_H is approaching 0. If $\|u_1^{k+1} - f_H^k\| > c_H^{k+1}$, we set $c_H^{k+1} = \|u_1^{k+1} - f_H^k\|$ and hence we have:

$$\mu_H^{k+1} = (\beta_{H1} / \sqrt{c_H^{k+1}})(\|Au_H^{k+1} + \gamma_H^{k+1} / \beta_{H1} - f_H\|_2^2 - 1 \tag{16}$$

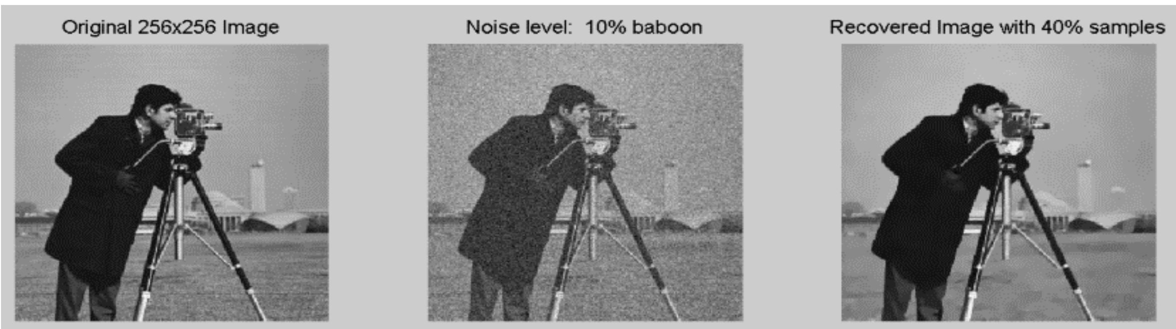
Similarly, two auxiliary variables u_2 and d_L are also needed to solve the sub-problem of u_L . Set $u_2 = Au_L$ and $d_L = Du_L$. In solving the sub-problem of u_L , the methods for solving u_L^{k+1} and u_2^{k+1} are similar to those for solving u_H^{k+1} and u_1^{k+1} . In other words, u_L^{k+1} and u_2^{k+1} can be obtained from u_H^{k+1} and u_1^{k+1} by replacing the corresponding subscript H with L , respectively. At the same time, d_L^{k+1} can be obtained in the following way different from d_H^{k+1} ; see (17) below.Given u_L^k, d_L^k , u_2^k, γ_L^k , δ_L^k and μ_L^k, and fix u_L^{k+1} and u_2^k , d_L^{k+1} can be expressed as :

$$d_L^{k+1} = \max\left(\sqrt{S^{k+1} * \overline{S}^{k+1}} - \frac{1}{\beta_{L2}}, 0\right) * \frac{1}{\sqrt{S^{k+1} * \overline{S}^{k+1}} - \frac{1}{\beta_{L2}}} \tag{17}$$

where $S^{k+1} = Du_L^{k+1} - \delta_L^k / \beta_{L2}$, and $\overline{S}$ denotes the conjugate of S .

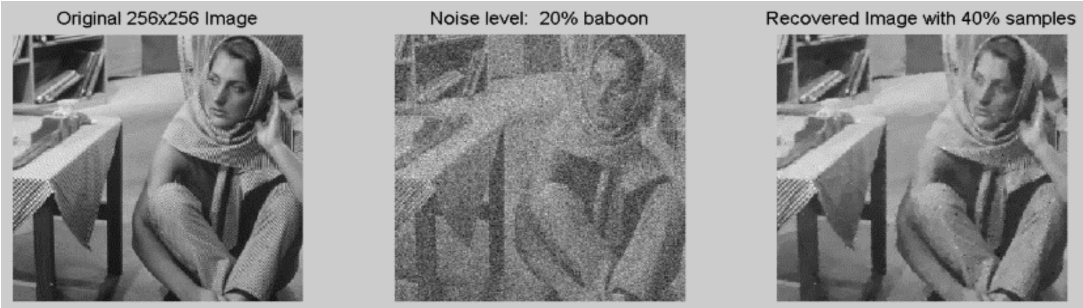
4. Experimental Results and Conclusion

Four classical digital images of 256 × 256 pixels: Lena, Cameraman, Barbara and Baboon are selected for the experiment. All the experiments were performed on MATLAB R2010b 7.11. The computer processor used in the experiment is Intel(R) i5-7300HQ, and RAM is 8.00GB. The noise is the Gaussian white noise with standard deviations of 0.05, 0.1, 0.2, and 0.3 respectively. The maximum number of iteration is set to 500; the algorithm precision tol is set as 10^{-4}, that is, the algorithm stops when $\|u^{k+1} - u^k\| / \|u^k\| < tol$. The penalty term parameters are set based on the experimental experience as $\beta_{H1} = 30$, $\beta_{H2} = 2*10^3$, $\beta_{L1} = 80$, $\beta_{L2} = 5$. SNR and SSIM are used as evaluation indexes for image reconstruction. The larger the index values of the two evaluations, the better. The unit of PSNR is dB, and the range of SSIM is from 0 to 1.The contrast experiment algorithms used in this work are the TV algorithm in reference[2] and compressed sensing algorithm in reference[4]. The compressed sensing algorithm and the proposed algorithm all use discrete wavelet transform (DWT) to sparse the noisy image. Gaussian random matrix is used for measurement matrix A , and the sampling ratio is 0.4.



(a) original image (b) noisy image (c) recovered image

Fig. 2: The image of cameraman de-noised by the proposed algorithm when the noise is 0.1.



(a) original image (b) noisy image (c) recovered image

Fig. 3: The image of barbara de-noised by the proposed algorithm when the noise is 0.2.

Table 1: Results of three de-noising algorithms

	Lena						Cameraman					
	TV		Reference[4]		Proposed algorithm		TV		Reference[4]		Proposed algorithm	
Noise	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
0.05	30.56	0.69	31.81	0.70	**32.94**	**0.72**	26.27	0.69	27.39	0.69	**28.5**	**0.72**
0.10	30.02	0.66	31.43	0.68	**32.85**	**0.70**	25.93	0.66	26.98	0.67	**28.04**	**0.69**
0.20	28.34	0.62	30.65	0.67	**32.50**	**0.68**	24.75	0.55	26.52	0.63	**28.01**	**0.65**
0.30	26.45	0.51	29.35	0.55	**31.28**	**0.63**	23.27	0.45	25.64	0.57	**27.57**	**0.63**
	Barbara						Baboon					
	TV		Reference[4]		Proposed		TV		Reference[4]		Proposed	
Noise	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM	PSNR	SSIM
0.05	23.65	0.61	26.51	0.68	**27.96**	**0.70**	27.75	0.65	29.96	0.69	**31.84**	**0.79**
0.10	23.52	0.59	26.23	0.66	**27.79**	**0.68**	27.54	0.64	29.88	0.66	**31.42**	**0.77**
0.20	22.99	0.57	25.84	0.61	**27.66**	**0.66**	26.80	0.62	29.31	0.62	**30.73**	**0.71**
0.30	22.16	0.53	25.08	0.57	**26.54**	**0.59**	25.76	0.59	28.41	0.61	**29.70**	**0.67**

It can be seen from Table 1 that for the de-noising effect of the four test images, the value of PSNR is more than 1 dB compared to the contrast experiment algorithms. Especially when the image noise is large, the SSIM value of the proposed algorithm is higher, indicating that the de-noised image has a good structural similarity with the original image. This is because different regularization is adopted for the texture and contour image according to the different features during the de-noising process, which allows better texture features of the image after de-noising.

The proposed algorithm is a compressed sensing de-noising algorithm based on the L1-L2 norm regularization. Firstly, the image is decomposed into a contour image and a texture image by the total variation spectral framework method, and then the contour image and the texture image are regularized according to the image structural features. Then, ADMM is adopted for solution. Finally, the experimental results demonstrate that the de-noising effect of the proposed algorithm is better than that of the contrast experiment algorithms. However, it is worth noting that the gradient descent algorithm used for solving sub-problems has a large number of iterative steps. In future research, the conjugate gradient algorithm with fewer iteration steps can be considered.

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