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The Problem of Traffic Signal Control Solved by LPLCC Model

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Abstract. This paper presents a linear program model with linear complementarity constraints (LPLCC) to solve traffic signal optimization problem. The objective function of the model is to obtain the minimization of total queue length with weight factors at the end of each cycle. Then, an algorithm based on SQP is proposed, by which local optimal solution can be obtained. Furthermore, three numerical experiments are proposed to study how to set the initial solution of the algorithm that can get a better local optimal solution more quickly. In particular, the results show that: The model is effective for different arrival rates and weight factors, and the lower the bound of the initial solution is, the better the optimal solution can be gotten.

Keywords: traffic signal control, optimization, weight factors, LPLCC.

1. Introduction

With the development of economy and the improvement of people's living standard, the number of private cars has been increasing sharply which results in a lot of congestion at traffic junctions. In particular, this phenomenon is more serious at peak time. A reasonable traffic signal control strategy can degrade jam, save time, reduce CO_2 emission and decrease the consumption of gasoline. This kind of problems have aroused a wide concern of scholars and many research achievements have been gotten (Yin, 2008; Motawej, et al., 2011; Liu, et al., 2013; Roshandeh, et al., 2014; Simões, et al., 2014; Ribeiro and Simões, 2012; Simões and Ribeiro, 2011; Simões and Ribeiro, 2016).

Compared to the previous work, these are four innovations in this paper as follows: (1) An modified algorithm based on SQP is used to deal with the model. (2) Three numerical experiments are proposed to study how to set the initial solution of the algorithm that can get a better local optimal solution more quickly. (3) A weight factor is introduced to present the vehicles queue length on each entrance lane because the importance of each entrance lane is different at crossroad during different period of a day. (4) The objective function of this paper is to get the minimization of the total queue length at the end of each cycle.

The remainder of this paper is organized as follows: In Section 2, the characteristic of the model is illustrated. In Section 3, the traffic intersection signal control model is formulated as an linear program model with nonlinear complementarity constraints. The numerical experiments are showed in Section 4 and the conclusion is arranged in Section 5.

2. Model Characterization

In this paper, we study the 4-phase system. Estimation of signal timing can be gotten by a methodology with the procedures illustrated here. The timing plan, including cycle lengths and green times, is determined by the objective of minimizing total queue length with weight factors at the end of determined cycles. The deliberate signalized junction has eight entrance lanes ($i=1,...,8$) and four phases ($l=1,2,3,4$) as Fig. 1 shows. The light cycle has three states: green, yellow and red. The time of each phase consists of green time, yellow time and clear time. In this study, the duration of the yellow time and the clearance time are fixed and

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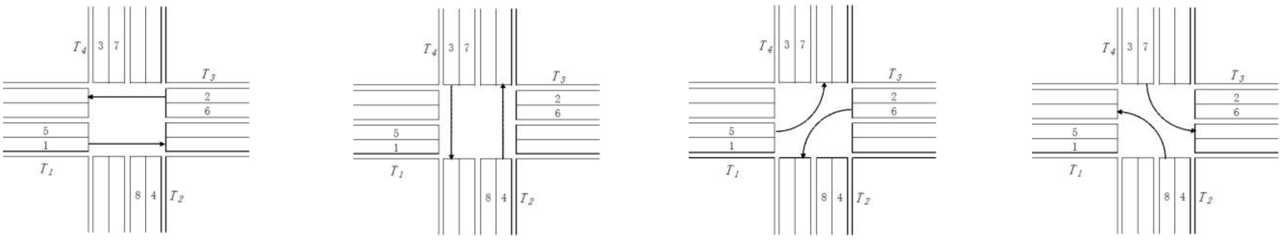
their values are denoted by d_Y and d_C, respectively. The conservation law of vehicles is expressed in formula (2.1).



(a) Phase 1 (b) Phase 2 (c) Phase 3 (d) Phase 4

Fig. 1: A typical 4-phase signal with left-turn protection control

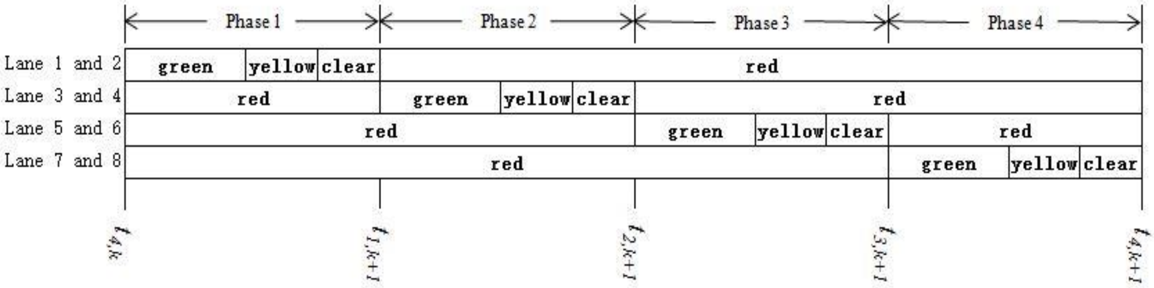


Fig. 2: Diagram of signal timing

$$x_{i,l,k+1} = x_{i,l-1,k+1} + E_{i,l,k+1} - D_{i,l,k} \tag{2.1}$$

In formula (2.1), $x_{i,l,k+1}$ and $x_{i,l-1,k+1}$ denote queue length in entrance lane i at the end of phase l and phase $l-1$ during the cycle $k+1$, respectively; $E_{i,l,k+1}$ and $D_{i,l,k+1}$ are the entrance vehicles length and departure vehicles queue in entrance lane i at the end of phase l during the cycle k, respectively, $L_{i,4,k} := L_{i,0,k+1}$.

Let $\lambda_{i,l,k+1}(t)$ and $\mu_{i,l,k+1}(t)$ denote the arrival rate and departure rate of lane i in phase l during cycle $k+1$ respectively. Then, $E_{i,l,k+1}$ and $D_{i,l,k+1}$ can be expressed as:

$$E_{i,l,k+1} = \int_0^{y_{G_l,k+1}} \lambda_{i,l,k+1}(t)dt, \qquad D_{i,l,k+1} = \int_0^{y_{G_l,k+1}} \mu_{i,l,k+1}(t)dt \tag{2.2}$$

More specifically, we assume that:

(1) $\bar{\mu}_i$ and $\bar{k}_i$ denote average departure rates when the traffic signal is green or yellow, respectively. $\mu_{i,k+1}(t) = \bar{\mu}_{i,k+1}$ and $k_{i,k+1}(t) = \bar{k}_{i,k+1}$, $i = 1,...,8$.

(2) The arrival rate $\lambda_{i,l,k+1}$ is determined by statistics approach in every phase of cycle, $\bar{\lambda}_{i,l,k+1} = \lambda_{i,l,k+1}(t)$.

(3) The right-turn movements are not considered in phase settings because of no conflictions with other vehicles movements.

Then, the formula (2.1) can be rewritten as:

$$x_{i,l,k+1} = \max\left\{\max\left\{x_{i,l,k-1} + \bar{\lambda}_{i,l,k+1} y_{G_{l,k+1}} - \bar{\mu}_{i,k+1} y_{G_{l,k+1}} \delta_{l,k+1}, 0\right\} + \bar{\lambda}_{i,l,k+1} d_Y - \bar{k}_{i,k+1} d_Y \delta_{l,k+1}, 0\right\} + \lambda_{i,l,k+1} d_C. \tag{2.3}$$

where $i = 1,...,8$, $k = 0,...N-1$, $x_{i,l-1,k} := x_{i,0,k+1}$,

$$\delta_{l,k+1} = \begin{cases} 1, & \textit{if light of lane i at phase l for cycle } k+1 \textit{ at intersection is green light;} \\ 0, & \text{otherwise.} \end{cases} \tag{2.4}$$

3. Problem Formulation

Therefore, the problem can be abstracted as the following model:

$$
\begin{aligned}
&\min \ \ \Upsilon^T x \\
&\text{s.t.} w = Qx + Py + q \\
&z = Hx + h \\
&m \le y \le M \\
&Gy = 0 \\
&x \ge 0 \\
&z^T w \le 0, z \ge 0, w \ge 0,
\end{aligned} \tag{3.1}
$$

where $x \in R^{32N}$, $\Upsilon \in R^{32N}$, $y \in R^{32N}$, $Q \in R^{32N\times 32N}$, $P \in R^{32N\times 32N}$, $q \in R^{32N}$, $H \in R^{32N\times 32N}$, $G \in R^{32N\times 32N}$, $h \in R^{32N}$, and $\varepsilon \ge 0$.

LPCC is a subclass of mathematical programs with equilibrium constraints (MPECs). The MPECs problem is known to be a very difficult problem because of it being non-smooth and non-convex in normal situation. Few methods that can calculate the global solution have been proposed so far.

Local convergence of sequence quadratic program (SQP) algorithm for mathematical programs with equilibrium constraints was proved by Fletcher R et al. (2006). Hence, we give an algorithm to solve the model (3.1) based on the SQP algorithm in this paper.

4. Algorithm and Numerical Experiments

4.1. Algorithm

We first sketch the modified algorithm procedure as follows:

Initialization: ε, $k=0$, x_0; Tolerances: $\varepsilon_{\min}$, $k_{\max}$; Inner iterations counter: $iter=0$; Problem information: A, b, Aeq, beq, lb, ub, $nonlcon$.

REPEAT

Step 1: Built the constraints.

Step 2: Built the MATLAB function:

$$[x, fval, exitflag, output, lambda, grad] = fmincon(fun, x_k, A, b, Aeq, beq, lb, ub, nonlcon, options).$$

Step 3: if $inter = k_{\max}$ or $\varepsilon \le \varepsilon_{\min}$, stop; otherwise go to step 4.

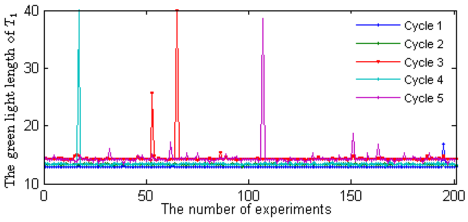
Step 4: update: $iter \leftarrow inter + output\ .iteration$, $x_{k+1} \leftarrow x$ and $\varepsilon : \varepsilon \leftarrow \varepsilon \times \rho$, $(0 < \rho < 1)$; go to step 2.

The 'fmincon' function is the body of this algorithm. As mentioned above, the SQP algorithm is a local convergence for linear programming with nonlinear complementarity constraints. This algorithm's convergence is obvious in this study.

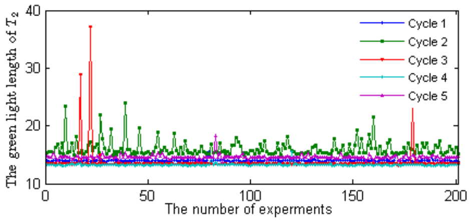
4.2. Numerical experiments

The LPLCC formulation is exploited and the SQP algorithm is used to highlight the usefulness of this methodology. All experiments are performed on an Intel(R) Core(TM) i5 CPU 2.6 GHz machine with 8 GB of RAM and software matlab2010b.

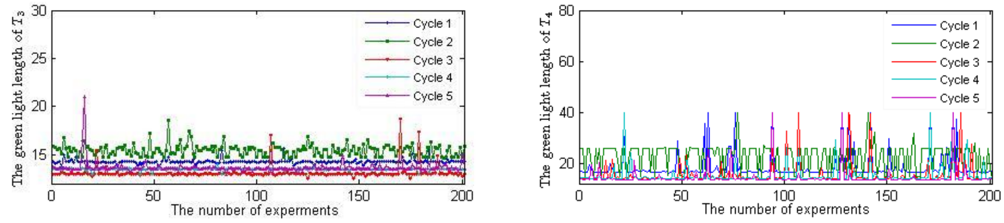
There are four aims of this subsection: (1) Obtain a better local optimal solution by doing experiments. (2) Prove the validation of the model for different arrival rates λ of different cycles. (3) Check efficiency of our model for different weight factors γ of different cycles. (4) Verify the efficiency of the algorithm and local convergence by numerical experiments.



(a) The green light length of T_1



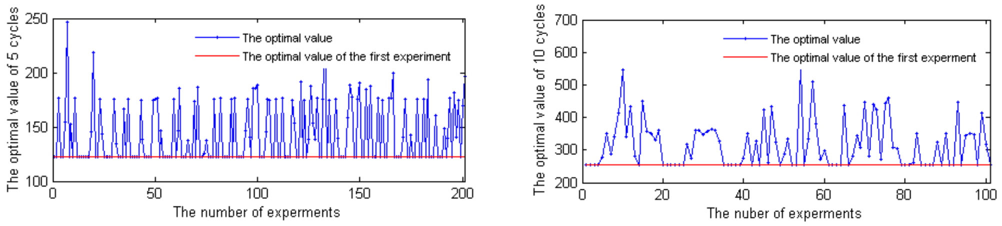
(b) The green light length of T_2



(c) The green light length of T_3 (d) The green light length of T_4

Fig. 3: The green light of T_1, T_2, T_3 and T_4 of every cycle

Let $d_Y = 3$ s, $d_C = 2$ s, $\bar{\mu} = 0.5$ veh/s and $\bar{k} = 0.2$ veh/s, minimum green light time is 7s and maximum green light time is 40s in all phases. In the Fig. 3 and Fig. 4, we give all parameters except initial solution in SQP algorithm.



(a) The optimal value of every experiment of 5 cycles (b) The optimal value of every experiment of 10 cycles

Fig. 4: The optimal value of every experiment of 5 and 10 cycles

In the first experiment, we set the lower bound of initial solution ($x = 0$ and $y = 7$). In other experiments, we get the initial value by random function. Fig. 3 presents that different initial solution results in different green light time T_1 (Fig. 3(a)), T_2 (Fig. 3(b)), T_3 (Fig. 3(c)) and T_4 (Fig. 3(d)) of cycle 1, cycle 2, cycle 3, cycle 4, and cycle 5.

Fig. 4(a) shows the optimal values based on different initial solutions. The blue line denotes different optimal value based on different initial solution (produced by random function except the first initial solution) while the red line presents the optimal value depending on the initial solution $x = 0$, $y = 7$. The results delicate that when let $x = 0$, $y = 7$, we can get a better local optimal value. The Fig. 4(b) presents the same results. Therefore, a conclusion can be drawn that the lower bound of x and y is, the better the initial solution can be gotten by modified algorithm.

We set arrival rate and weight factors produced by random function while other parameters are given. Especially, the initial solution of the modified algorithm is the lower bound x and y. The results show that different arrival rates or different weight factors can result in different optimal solution.

The Fig. 5 shows that the iteration times of modified algorithm is less when the value of the initial solutions is the lower bound value instead of random initial solutions.

As mentioned above, we can conclude that: (1) The model of this paper is effective. (2) The modified algorithm can calculate a better local optimal solution based on the initial solution when the value of variable is the lowest bound.

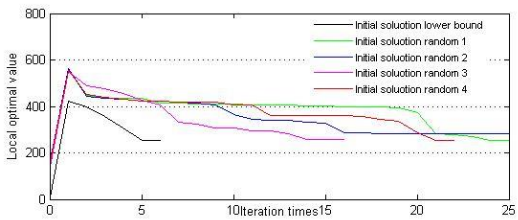


Fig. 5: The number of iterations of modified algorithm with different initial solutions

5. Conclusions

This paper proposes a optimization model for solving the optimal green time problem at single intersection. Through solving numerical experiments by modified algorithm, conclusions can be drawn as follows: (1) The objective function of this paper is reasonable. (2) The model is effective, that is, different arrival rates or diverse weight factors can contribute to different optimal green time by this model. (3) The lower bound of the initial solution of the algorithm can get a better local optimal solution.

Given its simple situation, more complex factors (such as stochastic arrival rate and multi-intersection) will be considered.

6. References

[1] Yin, Y. Robust Optimal Traffic Signal Timing. Transportation Research Part B: Methodological, 2008, 42(10): 911-924.

[2] Motawej, F., Bouyekhf, R., and Moudni, A. E. A note on Artificial Intelligence techniques and Dissipativity-based Approach in Traffic Signal Control For an Over-Saturated Intersection. IFAC Proceedings Volumes. 2011, 44(1), 10721-10726.

[3] Liu, X., Tan Z. F., and Chen, T. Optimization Methods of Signal Timing at Over-saturation Intersection and Empirical Study. Science Technology and Engineering. 2013, 13(9): 2604-2606.

[4] Roshandeh, A. M., Levinson, H. S., Li, Z., Patel, H., and Zhou, B. New methodology for intersection signal timing optimization to simultaneously minimize vehicle and pedestrian delays. Journal of Transportation Engineering. 2014,140(5), 04014009.

[5] Simões, M. L., and Ribeiro, I. M. Optimal signal timing signalized intersection by global optimization (OPT-I). 1st International Conference on Engineering and Applied Sciences Optimization, Proceedings. 2014:664- 679.

[6] Ribeiro, I. M., and M. L. Simões. Optimal Cycle for a Signalized Intersection Using Global Optimization and Complementarity. TOP: An Official Journal of the Spanish Society of Statistics and Operations. 2012, 20 (3): 779–790.

[7] Simões, M. L., and Ribeiro, I. M. Global Optimization and Complementarity for Solving a Semi-actuated Traffic Control Problem. Procedia: Social and Behavioral Sciences. 2011, 20: 390–397.

[8] Ribeiro, I. M., and Simões, M. D. L. D. O. The fully actuated traffic control problem solved by global optimization and complementarity. Engineering Optimization, 2016, 48(2), 199-212.