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Self-tuning Filter for High-Mobility Target Tracking System

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Abstract. Since the conventional self-tuning kalman filter can not deal with the filtering problem in the high-mobility target tracking system, we present a kind of modified self-tuning kalman filter. This filter is based on the jerk model and modern time series analysis method. Firstly, we use system identification method to estimate the noise statistic information, and then jerk model is used to generate matrix information which could be substituted into the kalman filter to adapt to the high-mobility environment. The parameters of the jerk model could be set to fit the kalman filter. The excellent convergence is proved by mathematical way, and a simulation example is given to illustrate the effectiveness of this filter.

Keywords: kalman filter, target tracking, self-tuning, jerk model

1. Introduction

Kalman filter is the most effective algorithm in the field of target tracking system, and it is especially applicable to the linear, discrete and finite dimensional systems [1]. In the filtering process, the parameters and noise statistics of the given system are needed to be exactly known [2]. But in the real environment, the parameters and noise statistics are always partly or completely unknown [3]. In order to solve the filtering problem with unknown parameters and noise statistics, Astrom and Witten [4] presented the self-tuning filters, which could get the estimations of the parameters and noise statistics by system identification method, and these estimations could be put into the optimal filters to get the self-tuning filter. Compared with the optimal filter, self-tuning filter is suboptimal. In the real applications, especially in the early stage of the filtering process, self-tuning method is necessary and feasible [4]. Deng [5] presented the Modern time series analysis method to get self-tuning filters, which rely on the autoregressive moving average(ARMA) model, system identification method, and dynamic error system analysis(DESA) method to design a kind of self-tuning filters which have good convergence to the optimal filters [5-8]. But, this kind of self-tuning filter can only deal with the situation when the target is under the low speed and mobility, and it can not work when the target is in high mobility condition. Kishore. Mehrotra and Pravas.R.Mahapatra [9] presented Jerk model which could work on the target tracking problem when the target is in high mobility condition, it provides us a kind of solution to conquer the filtering problem. This paper presents a kind of self-tuning kalman filter to deal with the high-mobility target tracking system with unknown parameters and noise statistics, which is based on the modern time series analysis method and jerk model. The self-tuning kalman filter converges to the optimal filter, and the proof is given by mathematical way. A simulation example is given to illustrate the effectiveness of the filter.

2. Problem Formulation

We consider about the time-invariant system which has the function equation(1), it also has the measurement equation(2)

$$X(t+1) = FX(t) + \Gamma w(t) \tag{1}$$

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$$y(t) = HX(t) + v(t) \tag{2}$$

where t is the discrete time, $X(t)$ is state vector, F is the state transforming matrix, $w(t)$ is the input noise of the system, it's variance can be remarked as Q. $y(t)$ is the measurement result of the ith sensor. H is the measurement matrix. $v(t)$ is the measurement noise of the sensor and it's variance can be marked as R.

Assumption 1 $w(t)$ and $v(t)$ are uncorrelated noises, they have the relationship

$$E\{\begin{bmatrix} w(t) \\ v(t) \end{bmatrix}\begin{bmatrix} w^{\mathrm{T}}(t) \; v^{\mathrm{T}}(t) \end{bmatrix}\} = \begin{bmatrix} Q & 0 \\ 0 & \mathrm{R} \end{bmatrix} \tag{3}$$

where E denotes the mathematical expectation, T is the transpose.

Assumption2 $y(t)$ is bounded, there existing a constant c that satisfies $|y(t)| \le c$.

Assumption3 Q and R are both unknown.

3. Self-tuning Kalman Filter

Lemma 1 Optimal Kalman filter

For the system with known noise and parameters, the optimal Kalman filter can be substituted by the formula (4)-(9)

$$P\langle t|t-1\rangle = FP\langle t-1|t-1\rangle F' + \Gamma Q \Gamma' \tag{4}$$

$$P\langle t|t\rangle = (I - K(t)H)P\langle t|t-1\rangle \tag{5}$$

$$K(t) = P\langle t|t\rangle H'(HP\langle t|t-1\rangle H' + R)^{-1} \tag{6}$$

$$\hat{x}(t|t-1) = F\hat{x}(t|t) \tag{7}$$

$$\varepsilon(t) = y(t) - H\hat{x}(t|t-1) \tag{8}$$

$$\hat{x}(t|t) = \hat{x}(t|t-1) + K(t)\varepsilon(t) \tag{9}$$

The formula(4)-(9) are the classical Kalman filtering process. In every step t, the prediction error variance matrix $P\langle t|t-1\rangle$ satisfies the Riccati function

$$P\langle t|t-1\rangle = F[P\langle t-1|t-2\rangle - P\langle t-1|t-2\rangle H^{\mathrm{T}}(HP\langle t-1|t-2\rangle H^{\mathrm{T}} + R)^{-1}HP\langle t-1|t-2\rangle]F^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} \tag{10}$$

When the noises are unknown, in every step t, the noise variance Q and R can be estimated by system identification method. The estimations of Q and R at step t are remarked as $\hat{Q}(t)$ and $\hat{R}(t)$ (t could be omitted for simplicity). Substituting (1) to (2), (2) could be converted into Autoregressive Moving Average(ARMA) model by mathematical means, so we can get

$$A(q^{-1})x(t) = B(q^{-1})m(t) \tag{11}$$

$A(q^{-1})$ and $C(q^{-1})$ are the polynomial with the form

$$X(q^{-1}) = 1 + x_1 q^{-1} + \ldots + x_n q^{-n} \tag{12}$$

q^{-1} is the unit delay factor, $q^{-1}x(t+1) = x(t)$. For the ARMA model, In general we can use the extended forms of the Recursive Least Squares(RLS) methods, such as extended least squares method or recursive instrument variable method to get the variance of the noises. The estimations of Q and R are substituted into (4) and (6), we can get

$$\hat{P}\langle t|t-1\rangle = F\hat{P}\langle t-1|t-1\rangle F' + \Gamma\hat{Q}\Gamma' \tag{13}$$

$$\hat{K}(t) = \hat{P}\langle t|t\rangle H'(H\hat{P}\langle t|t-1\rangle H' + \hat{R})^{-1} \tag{14}$$

The results of (13) and (14) are substituted into (9), we can get the self-tuning filter

$$\hat{x}^s(t|t) = \hat{x}^s(t|t-1) + \hat{K}(t)\hat{\varepsilon}(t) \tag{15}$$

When the system is in the high mobility state, F has the relationship

$$\lim_{\alpha\to 0} F = \begin{pmatrix} 1 & T & T^2/2 & T^3/6 \\ 0 & 1 & T & T^2/2 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{16}$$

When the value of α is close to infinity, the system satisfies uniformly accelerated motion model, F can be expressed as

$$\lim_{\alpha\to\infty} F = \begin{pmatrix} 1 & T & T^2/2 & 0 \\ 0 & 1 & T & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{17}$$

the setting of the value of α is depending on the expert's knowledge and experience, when $\alpha = 1/20$, the target is in turning state. When $\alpha = 1/60$, the target is in escaping state. When $\alpha = 1$, the target is under atmospheric disturbance. Take α into F, the corresponding ARMA model could be obtained. (2) could be substituted into the state equation(1), we can get the ARMA model

$$\det(I - Fq^{-1})y(t) = Hadj(I - Fq^{-1})\Gamma w(t-1) + \det(I - Fq^{-1})v(t) \tag{18}$$

We can define

$$r(t) = \det(I - Fq^{-1})y(t) \tag{19}$$

The correlation function of $r(t)$ could be obtained by sampling method

$$\hat{R}_r^t(k) = \frac{1}{t}\sum_{u=1}^{t} r(u)r(u-k) \tag{20}$$

The right side of the formula (11) is a stationary random sequence, (11) can be expressed as the form of a moving average model. correlation method is used to get the linear equation group

$$\hat{R}_r^t(k) = \sum_{i=k}^{n_m} m_i Q m_{n_m - i} + \sum_{i=k}^{n_n} n_i R n_{n_n - i}, k = 0,1,\ldots,n_0, n_0 = \max(m_m, m_n) \tag{21}$$

The formula(19) is a group of linear equations with the unknown variables Q and R. When this equations are compatible, the estimations $\hat{Q}(t)$ and $\hat{R}(t)$ at step t could be obtained. Substituting $\hat{Q}(t)$ and $\hat{R}(t)$ into (19), the self-tuning kalman filter is obtained.

4. Convergence Analysis

According to the ergodicity of the correlation function and the continuity of the elementary mathematics, There has the relationship

$$\hat{Q}(t) \to Q, \hat{R}(t) \to R, t \to \infty, w.p.1 \tag{20}$$

$w.p.1$ is short for "with possibility 1". The estimations are taken into the self-tuning, DESA method is used to prove the self-tuning filter could converge to the optimal filter in a realization. There has the relationship

$$\hat{x}^s(t|t) \to \hat{x}(t|t), t \to \infty, i.a.r \tag{21}$$

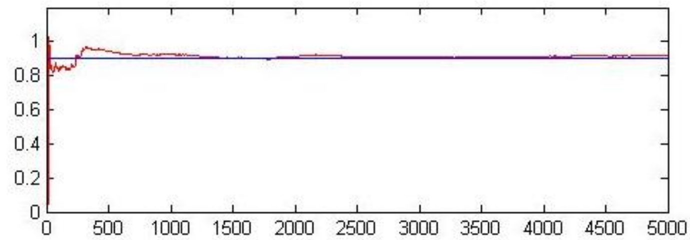
$i.a.r$ is short for "in a realization". The convergence of $i.a.r$ is weaker than $w.p.1$.

5. Simulation

Question: For the target-tracking system(1) and (2), when the variance of the input noise and measurement noise are unknown, how could we get the self-tuning kalman filter?

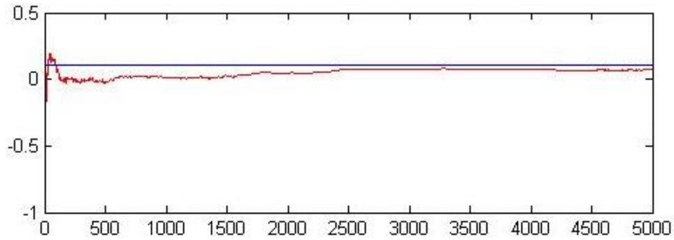
In the simulation we take the transpose $T = 0.5s$, when the target is under atmospheric disturbance, $\alpha = 1$. The real value of the variance of the input noise $\sigma_w^2 = 0.9$. The real value of the variance of the measurement noise $\sigma_v^2 = 0.2$.

The algorithm presented by this paper is used to get the self-tuning filter. Matlab is used to simulate this question. We can get the figures below. Figure1 says the relationship between the estimation and the real value of the input noise variance. Figure2 says the relationship between the estimation and the real value of the measurement noise variance. In these figures, the curve says the estimation, the straight line says the real value. Figure3 says the relationship between the self-tuning and optimal kalman filter. In this figure, the curve says the optimal filter, the spot says the self-tuning filter.



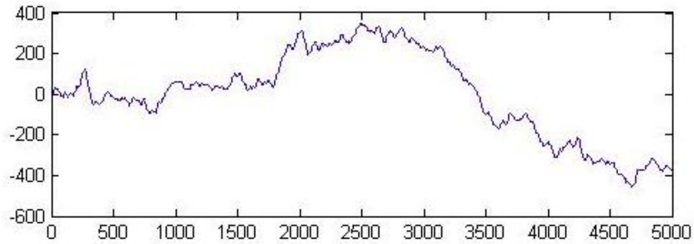
$\hat{\sigma}_w^2$ σ_w^2

Fig. 1: The estimations of the variance of the input noise.



$\hat{\sigma}_v^2$ σ_v^2

Fig. 2: The estimations of the variance of the measurement noise.



Self-tuning filter Optimal filter

Fig. 3: Self-tuning and optimal kalman filter.

6. Conclusion

In this paper, we consider about a high-mobility system, when the parameters and the noise statistics are unknown, a kind of self-tuning filter is presented, and its convergence is proved by mathematical method. A simulation example is given to illustrate the effectiveness of the algorithm. In these simulation figures, we can see that the estimations have good convergence with the real values. We can also see that the self-tuning filter and the optimal filter are in good agreement. These simulation results show that self-tuning theory which rely on the modern time series theory and system identification theory could deal with the high-mobility target tracking problem.

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8. References

[1] Ran C J, Deng Z L. Information fusion multi-stage identification method for multisensor multi-channel ARMA models[C]. In: Proceedings of the International Conference on Mechatronics and Automation.Beijing,China:IEEE, 2011:2216-2221.

[2] Zili Deng, Yuan Gao, Chunbo Li, Gang Hao, “Self-tuning decoupled information fusion wiener state component filters and their convergence”, Automatica,vol.44,no.3,pp.685-695,2008.

[3] Gao Y, Ran C J, Sun X J, et al. Optimal and self-tuning weighted measurement fusion Kalman filters and their asymptotic global optimality. International Journal of Adaptive Control and Signal Processing, vol.24, no.1, pp.982-1004, 2010.

[4] K. J. Astrom, B. Wittenmark, “On self tuning regulators”, Automatica, 1973, 9:185-199.

[5] Ran C J, Deng Z L. Self-tuning weighted measurement fusion Kalman filter and its convergence.Journal of Control Theory and Applications, vol.8, no.4, pp.435-440, 2010.

[6] Deng Z L, “Self-tuning α-β-γ Tracking Filter”, Control and Decision, vol.6, no.5, pp.384-387, 1992.

[7] Deng Z L, “Self-tuning α-β Tracking Filter”, ACTA Automatica SINICA, vol.18, no.6, pp.720-723, 1993.

[8] P. J. Dua, S. C. Liu, J. S. Xia and Y. D. Zhao, “Information fusion techniques for change detection from multi-temporal remote sensing images”, Information Fusion, vol. 14, no. 1,pp. 19-27,2013.

[9] Kishore Mehrotra, Pravas R.Mahapatra, “A Jerk Model for Tracking Highly Maneuvering Target”, IEEE Transactions on Aerospace and Electronics Systems,vol.33, no.4, pp.1094-1105, 1997.