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Missing Data Imputation Approach Based on Adaptive Compressed Sensing

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Abstract. The paper presents a missing data imputation method based on compressed Sensing (CS). First of all, the problem of data imputation is translated into the recovery of sparse vector under the framework of compressed sensing. Secondly, we propose an improved greedy reconstruction algorithm called Double Try Sparsity Adaptive Matching Pursuit (DTSAMP). The algorithm obtains the estimation of sparsity by trying twice to approximate the value of sparsity, and then approximates the estimated value in each iteration. As a result, the missing data sets can be reconstructed without prior information of the sparsity. Furthermore, the step size and support set are well controlled by setting thresholds during the iteration. The simulation results show that the proposed algorithm is superior to other methods in terms of reconstruction speed and accuracy, as well as better robustness.

Keywords: data imputation, compressed sensing, sparsity adaptive, step length control, maching pursuit.

1. Introduction

With the development of science and technology, the collection and use of data has become the cornerstone of modern society. Especially in recent years, due to the widespread use of sensors, a new digital society has been created on the basis of massive data, and the importance of data has become increasingly prominent. In the process of data transmission, data loss is very common due to various reasons, such as hardware failure, channel fading, channel conflict and line blocking [1].

The preprocessing of data sets with missing data is an important task before data mining. If missing values in data sets cannot be imputed accurately, many existing data mining methods will be useless. When there are few missing values in the data set, direct deletion is always used to complete the processing work. However, deletion will result in the loss of information if the number of missing values is quite large. Imputing the missing values is another way to reconstruct the data sets with missing data, statistical methods or machine learning methods are usually used to complete the work in practice [2].

At present, there have been a large number of methods to complete the estimation of missing values, different imputation methods have their own advantages and disadvantages. Linear interpolation is one of the simplest data imputation methods, but it does not work well when dealing with continuous data loss [3]. The K-nearest Neighbor method (KNN) does not need to predict the quantitative or qualitative properties of the missing values in advance. This method can directly process multiple missing values, however, it performs poorly when running on large data sets [4]. Rough set theory is an effective way to deal with the problem of uncertainty, on the other hand, it cannot complete parameter optimization and missing data classification, which results in low precision of data completion [5]. Random forest (RF) can process high-dimensional data with high accuracy, while it has a large computational cost when processing a large amount of data [6]. Support vector machine (SVM) is insensitive to outliers and has high robustness, but this algorithm is of high computational complexity [7]. Neural network has excellent performance, nevertheless, it requires a huge training data set and is prone to over-fitting [8]. The tensor factorization method has advantages in high-dimensional data imputing, and makes full use of the implicit information between data of different

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dimensions, yet it also has problems in calculation [9, 10]. These data imputation methods are effective in their suitable cases, but not for data sets with a large amount of missing data.

Compressed sensing is a novel sampling theory, which can reconstruct the original signal on the basis of a small number of samples [11]. According to Nyquist sampling theorem, in order to retain the information in the original signal completely, the sampling frequency must be greater than 2 times of the highest frequency. The compressed sensing method breaks through the limitation of sampling principle, and can complete signal sampling with much less than 2 times of the maximum frequency, which greatly reduces the number of sampling. According to the sparse characteristics of the signal, the original signal can be recovered by solving a nonlinear optimization problem, and only a small number of data values are needed. The sparseness of the signal is an important foundation of the compressed sensing theory. In a sparse signal, a large number of signals have not been collected, which is very similar to the situation where the data set contains a large number of missing data. Therefore, we can also treat the missing data set as a sparse vector and reconstruct the data set with a small amount of data. To sum up, the compressed sensing theory can be used to solve the interpolation problem of data sets with large amounts of missing data.

The design of reconstruction algorithm is one of the keys to data reconstruction in compressed sensing system, which determines the efficiency to a large extent. The existing reconstruction algorithms mainly include convex-optimization algorithms, greedy algorithms, Bayesian algorithms, noniterative reconstruction algorithms and machine learning algorithms [12]. Greedy algorithms are widely used because of their low reconstruction complexity. When the sparsity is known, greedy algorithms such as matching pursuit (MP) [13], orthogonal matching pursuit (OMP) [14], regularized orthogonal matching pursuit (ROMP) [15], stagewise orthogonal matching pursuit (StOMP) [16], compressive sampling matching pursuit (CoSaMP) [16] and subspace pursuit (SP) [17] could accurately reconstruct the results. But this piece of information may not be available in practical applications, the sparsity adaptive matching pursuit (SAMP) [18] is proposed to recover the signal when the sparsity is unknown. This algorithm needs a trade-off between iterative speed and exact recovery, but it is difficult to keep a balance between them.

In order to solve this problem, a large number of algorithms have been proposed. Iterative regularization sparsity adaptive matching pursuit (IR-SAMP) algorithm [19] use regularization to achieve retrospective screening, so that the inappropriate atoms can be removed. Filtering-based regularized sparsity variable step-size matching pursuit (FRSVssMP) algorithm [20] propose two stages of variant step sizes to approximate the true sparsity. Bidirectional sparsity adaptive adjustment and weak selection of atoms (BSA-WSAMP) algorithm [21] utilize the estimated value of the energy difference between the adjacent two iterations to decide the variable step size. Sparsity adaptive greedy iterative (SAGI) algorithm [22] introduce a new sparsity pre-estimation strategy, whereby the estimated sparsity can be easily obtained. However, the insufficient compatibility between accuracy and efficiency of these algorithms still need to be improved.

According to the above analysis, this paper proposed a missing data imputation method based on compressed sensing, which transformed the problem of missing data imputation into a sparse signal recovery problem. Furthermore, a reconstruction algorithm was proposed, which improved the operation efficiency by means of sparsity estimation and variable step size. What's more, it also improved the reconstruction accuracy by the control of support set and the introduction of backtracking. The simulation results indicated that the algorithm proposed can well reconstruct the data set even when the sparse conversion effect is very poor. It also had better robustness and greater advantages in reconstruction accuracy and running speed.

2. CS Theory Framework

Suppose that x is a sampled length-N signal, y is an M -dimensional vector that consists of linear projections of the vector x, If the number of nonzeros in x is much smaller than N, it is called K -sparse. This encoding system can be described as follows:

$$y = \mathbf{\Phi} x, \tag{1}$$

where $\mathbf{\Phi}$ represents an $M \times N$ sampling matrix.

In this theory, we assume that x is considered to be K -parse, whereas signals may not always be sparse in practical applications. To solve the problem, we can transform the signal to another set of bases so that the signal is sparse on this set of bases, which is called the sparse representation of the signal:

$$x = \mathbf{\Psi}\theta, \tag{2}$$

where $\mathbf{\Psi}$ is an $N \times N$ transform basis, and θ is an N-dimensional signal with no more than K nonzero coefficients.

In general, if the values except the K nonzero coefficients are small, we can also consider this vector to be K-sparse, then:

$$\mathrm{y} = \mathbf{\Phi}x = \mathbf{\Phi\Psi}\theta = \mathbf{A}\theta, \tag{3}$$

where $\mathbf{A = \Phi\Psi}$ is an $M \times N$ sampling matrix.

After receiving the signal, the receiver needs to complete the signal reconstruction. According to elementary linear algebra, x cannot be uniquely recovered from y by linear algebraic means. To solve the problem, sparsity is supposed to be a powerful constraint. Assuming that x is K -sparse, it can be well reconstructed if $\mathbf{\Phi}$ satisfies the Restricted Isometry Property (RIP) as follows:

$$\left(1-\delta_K\right)\|x\|_2^2 \le \|\mathbf{\Phi}x\|_2 \le \left(1+\delta_K\right)\|x\|_2^2, \tag{4}$$

where δ_k is the RIP constant。

3. Double Try Sparsity Adaptive Matching Pursuit

3.1. Pre-estimation of Sparsity

A sparsity underestimation method has been proposed in literature [22]. The idea is to obtain a support set Γ_c through the matching test, which is slightly smaller than the real support set Γ, and the obtained sparsity estimate K_c will be slightly smaller than the true sparsity K .

Assume the sensing matrix $\mathbf{\Phi}$ satisfies the RIP property with parameter $\left(K,\delta_k\right)$ and $K_c \ge K$. Since δ_K is monotonic, we have $\left\|\mathbf{\Phi}_{\Gamma_c}^T y\right\|_2 \ge \left(1-\delta_{2K}\right)/\sqrt{1+\delta_{2K}} \cdot \|y\|_2$. The theorem is sufficient and unnecessary, and its negation proposition is usually used to determine the size of the sparsity estimation value, that is, if there is $\left\|\mathbf{\Phi}_{\Gamma_c}^T y\right\|_2 < \left(1-\delta_{2K}\right)/\sqrt{1+\delta_{2K}} \cdot \|y\|_2$, then $K_c < K$.

The premise of the theorem is that the support set obtained by the matching test is accurate. However, if the wrong column is selected in the support set, the sparsity estimate will be inaccurate. Reference [22] select the support set by calculate the inner products of the residual with the columns of $\mathbf{\Phi}$. Experiments show that the estimated values obtained by using this method have significant instability.

In this paper, the backtracking idea in the SP algorithm is introduced to improve the selection method of the support set, which improves the accuracy of the pre-estimation and reduces the probability of introducing wrong candidates in the pre-estimation process. First, calculate the inner product of the residual and each column of the sampling matrix, select the candidates corresponding to the first k maximum values, and establish a preselected set as $S_k = Max(\left|\mathbf{\Phi}^T r_{k-1}\right|, k)$. In each iteration, the support set F_{k-1} obtained from the previous iteration is combined with the preselected set S_k of this iteration to obtain a candidate set $C_k = F_{k-1} \bigcup S_k$. Finally, the candidate set is tested, and the candidates corresponding to the first k maximum absolute value was selected as the new support set of this round, which is chosen with the least square method as $F_k = Max(\left|\mathbf{\Phi}_{C_k}^{\dagger} y\right|, k)$, and the residual of the round was calculated as $r_k = y - \mathbf{\Phi}_F \mathbf{\Phi}_F^{\dagger} y$.

In order to improve test efficiency, exponential iteration is used to test sparsity. The sparsity estimation function is defined as $k = a \bullet b^m (m = 1, 2, \cdots)$, and the value of m gradually increases. When $\left\|\mathbf{\Phi}_k^T y\right\|_2 < \left(1-\delta_{2K}\right)/\sqrt{1+\delta_{2K}} \cdot \|y\|_2$ is encountered, it means that k has not reached the critical value, and the next attempt is started. The iteration is stopped until it meets $\left\|\boldsymbol{\Phi}_k^T y\right\|_2 > \left(1-\delta_{2K}\right)/\sqrt{1+\delta_{2K}} \cdot \left\|y\right\|_2$, and the value of k obtained in the last iteration is taken as the sparsity estimation value.

3.2. Re-estimation of Sparsity

Reference [23] gives a criterion for sparse overestimation. If there is $\left\|\boldsymbol{\Phi}_k^T y\right\|_2 > \left(1+\delta_{2K}\right)/\sqrt{1-\delta_{2K}} \cdot \left\|y\right\|_2$, then $k > K$.

We improves the selection method of support set in its sparsity estimation, the backtracking idea in SP algorithm is also introduced to reduce the probability of wrong matching and improve the accuracy of reconstruction. The re-estimation of sparsity is based on the pre-estimation of sparsity. According to the experimental results, when the sparsity is greater than $M/2$, the reconstruction is hard to succeed, so we can initialize $K_{\mathrm{H}}=M/2$ and record the pre-estimated value of sparsity as K_{L}. Then the dichotomy method is used to estimate the sparsity between K_{L} and K_H and the sparsity $k=\left(K_H+K_L\right)/2$ is used for trial. If $\left\|\boldsymbol{\Phi}_k^T y\right\|_2 > \left(1+\delta_{2K}\right)/\sqrt{1-\delta_{2K}} \cdot \left\|y\right\|_2$, there is $k>K$, and $K_H=k$ is assigned. On the other hand. If $\left\|\boldsymbol{\Phi}_k^T y\right\|_2 < \left(1-\delta_{2K}\right)/\sqrt{1+\delta_{2K}} \cdot \left\|y\right\|_2$, there is $k<K$, and $K_L=k$ is assigned at this time. Repeat the process, and the final value is the estimate. Experiments showed that the sparsity re-estimation step improved the reconstruction probability.

3.3. Iterative Control

Iterative step introduces the idea of backtracking and all candidates are evaluated at each iteration. In order to improve the iterative speed of the algorithm, a weak matching mechanism is adopted in the selection of the pre-selected set, and only the candidates exceeding the threshold value are selected to join the set. The selection conditions are as follows:

$$\left|\left\langle \varphi_j,\ r^{k-1}\right\rangle\right| \geq \xi \underset{l\in[1,k]}{Max}\left|\left\langle \varphi_l,\ r^{k-1}\right\rangle\right|, \tag{5}$$

where φ_i is the i-th element of $\boldsymbol{\Phi}$, r^{k-1} is the residual corresponding to iteration $k-1$ in the iterative process, and ξ is the filtering parameter.

Setting a smaller ξ allows you to select more candidates per search, which speeds up matching. However, it also increases the chance of picking up the wrong candidates. As the iteration goes on, if ξ is fixed, the possibility of selecting the wrong candidate will increase. The wrong candidate will affect the residual, thus affecting the accuracy of reconstruction. Therefore, the value of ξ needs to be controlled to reduce the possibility of false matches. Set a threshold $\varepsilon_1=\left\|y\right\|_2/n$, and when $\left\|r\right\|_2$ is greater than the matching threshold ε_1, a smaller ξ is adopted to speed up the residual matching. When $\left\|r\right\|_2$ is smaller than the matching threshold, the selection parameter ξ is gradually increased to reduce the number of selected candidates and improve the matching accuracy. Here, the value of parameter ξ adopts the Linear monotonically increasing form:

$$\xi=\begin{cases} \xi & \left|r^{k-1}\right| > \varepsilon_1 \\ \xi+l & \left|r^{k-1}\right| \leq \varepsilon_1 \end{cases}. \tag{6}$$

Through the reconstruction experiment of standard normal random numbers with different sparsity, it is found that the l_2-norm of residuals presented some certain rules with the increase of iteration times. As can be seen from Fig. 1, the l_2-norm of residual decreases gradually with the increase of iterations. In order to improve the reconstruction speed and optimize the parameter setting, threshold ε_1 is also adopted as the cut-off point. When the residual's norm $\left\|r\right\|_2$ is greater than the threshold, the reconstruction speed is improved by using large step length. Otherwise, the small step is adopted to improve the reconstruction accuracy.

As to the $K=100$ curve in Fig. 1, the l_2-norm of residual rebounded when the number of iterations reached 50 due to the introduction of wrong candidates in the iteration process. Because of the backtracking mechanism, the curve drops rapidly after eliminating the wrong candidates. This phenomenon seldom occur when the sparsity is small, but it often occurs when the sparsity is large. Therefore, in the case of good sparse conversion effect, using $\left\|r^{(k)}\right\|_2 > \left\|r^{(k-1)}\right\|_2$ as the halting condition can avoid parameter setting and has little influence on reconstruction results. Otherwise, we set $\left\|r^{(k)}\right\|_2 < \varepsilon_2$ to stop the iteration, which can improve the reconstruction success rate of the algorithm in the case of poor sparsity.

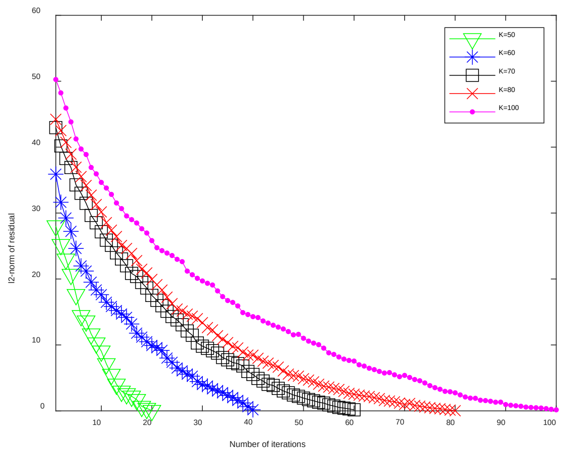


Fig. 1: Variation of l_2-norm during iteration.

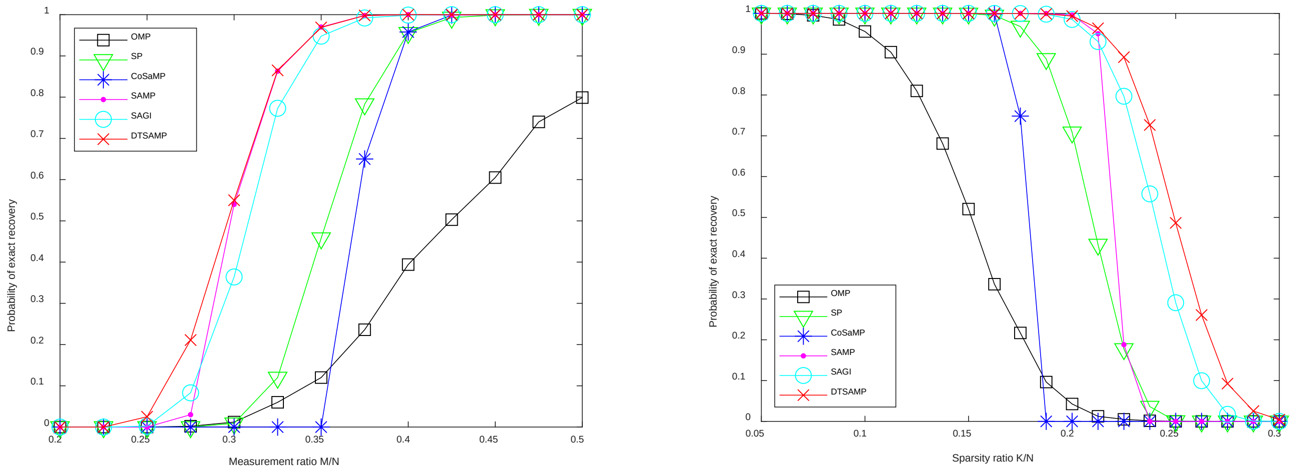
4. Simulation Results

In this section, series of experiments are conducted to compare the simulation results of the proposed algorithm with other greedy algorithms. As a reconstruction algorithm, the recovery accuracy and recovery time are important parameters to measure the effect of the algorithm. To better evaluate the algorithm, the recovery success rate and recovery time of the proposed algorithm are compared with other algorithms later. Furthermore, the algorithm is tested on a real dataset.

4.1. Success rate of Reconstruction

By comparing with OMP, SP, CoSaMP, SAMP, SAGI [22] and other algorithms, the performance of DTSAMP algorithm can be verified. We define the value of M/N as the measurement ratio, that is, the proportion of known data to all data. The reconstruction probabilities of the above algorithms are compared under different measurement ratios. In the reconstruction experiment, the original data x and estimated data $\hat{x}$ are compared. If $\left\|x-\hat{x}\right\|_2 < 10^{-6}$ is satisfied, the reconstruction is regarded as successful. In the first experiment, we fixed the sparsity, changed the measurement M, and observed the reconstruction probability of various algorithms under different measurement ratios M/N.

Fig. 2 (a) shows that the reconstruction probability of the algorithm increases with the increase of measurement ratio, but different measurement ratios have different effects on different algorithms. Compared with other algorithms, DTSAMP algorithm can reconstruct the missing data set at a lower measurement ratio. Especially when the measurement ratio is 0.25, only this algorithm can achieve reconstruction, so it has greater advantages at low measurement ratio.



(a) (b)

Fig. 2: (a) Prob. of exact recovery vs. the measurement ratio M/N. In this experiment, $N = 400$, $K = 50$. (b) Prob. of exact recovery vs. the sparsity ratio K/N. In this experiment, $N = 400$, $M = 200$.

Different from the laboratory, the sparsity of data is often unknown in practice. In some cases, the sparse matrix cannot complete the sparse representation of data well. Therefore, in order to evaluate the performance of missing data reconstruction, we also need to compare the adaptability of reconstruction algorithm to different sparsity under the same measurement value. In this experiment, we fixed the measurement, changed the sparsity K, and observed the reconstruction probability of various algorithms under different sparsity ratios K/N.

As can be seen from Fig. 2 (b), when the sparse ratio is greater than 0.275, only the proposed algorithm can complete the reconstruction of missing data. It can be seen that DTSAMP has a higher tolerance for sparsity when the number of measurements is fixed. With the same number of measurements, the missing data can be reconstructed with better robustness in a larger range of sparsity.

The proposed algorithm is able to offer great advantages in reconstruction success rate over others, mainly due to its improvement in the estimation phase. As the backtracking idea in the SP algorithm is introduced in the estimation phase to enhance the support set selection, the accuracy of the estimation step is improved and the likelihood of introducing false candidates is reduced.

4.2. Time of Reconstruction

The computational complexity of different algorithms is compared below, and the average reconstruction time of various algorithms under different sparsity conditions is calculated with fixed measured values. According to the conclusion in Fig. 2 (b), some algorithms cannot complete the reconstruction of missing data sets when the sparsity is relatively high. In order to compare the reconstruction time, we only intercept the reconstruction time image when the sparsity ratio is less than 0.175. As shown in Fig. 3, when the sparsity of DTSAMP algorithm gradually increases, the reconstruction time does not show a steep upward trend, and the time curve is generally flat. At the same time, as the sparsity ratio increases, the reconstruction time curve of other algorithms becomes steep, and their reconstruction time quickly exceeds the DTSAMP algorithm. The proposed algorithm employs an estimation strategy that tamps the foundation of the initial iteration. It applies adaptive variable step size, which reduces the number of iterations considerably. The introduction of a weak matching mechanism reduces the time required for a single iteration. Therefore, greatly increases the running speed of the algorithm.

Although SP and CoSaMP take less time under low sparsity, but they need to know the value of sparsity K in advance. The proposed algorithm spend least time among the algorithms wihout knowing the sparsity. Therefore, this algorithm has great advantages in adaptability and computational complexity.

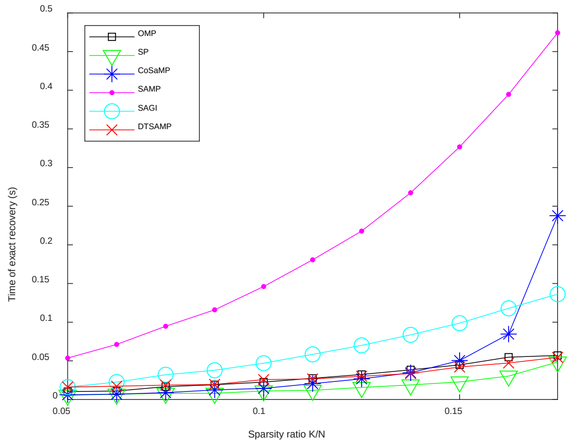
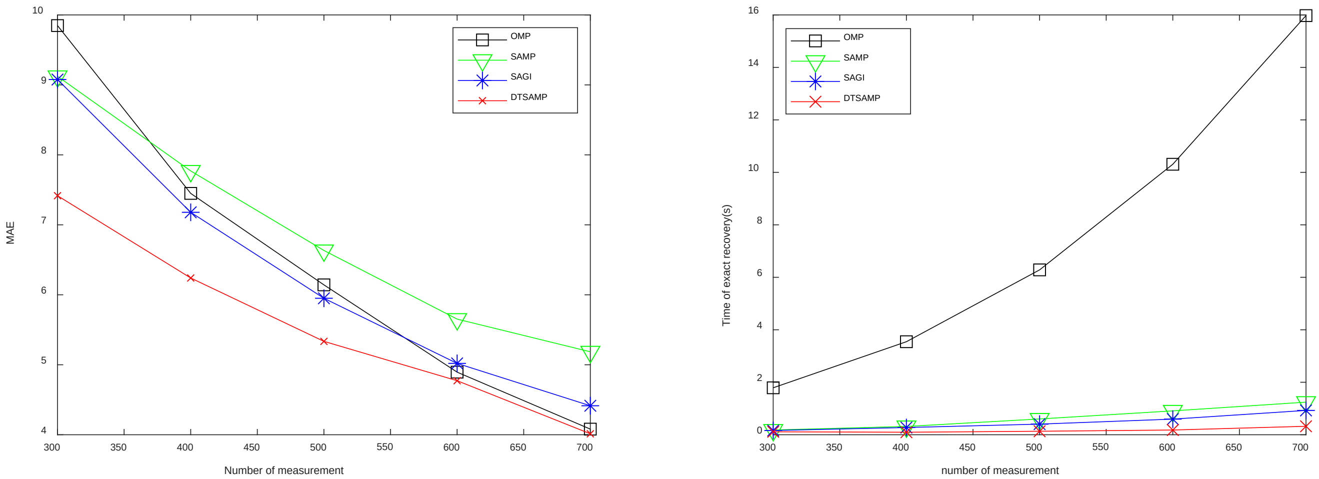


Fig. 3: Time of exact recovery vs. the sparsity ratio K/N. In this experiment, $N = 400$, $M = 200$.

4.3. Real Dataset Test

The FiveCitiePMData data set provides the PM2.5 data in Beijing, Shanghai, Guangzhou, Chengdu and Shenyang. Meanwhile, meteorological data for each city are also included. A total of 1416 humidity data in January and February of 2014 in Shanghai are selected. The DCT is chosen as the sparse representation basis, and the Boolean random matrix is used as the observation matrix. The value range of is [300,700].



(a) (b)

Fig. 4: (a) Mean absolute error (MAE) vs. the number of measurement M.
(b) Time of exact recovery vs. the number of measurement M.

As can be seen from Fig. 4, the mean absolute error (MAE) of the reconstructed data set decreases with the increase of the measurement values, since the amount of information obtained also increases with the increase of the number of measurements. However, DTSAMP algorithm has the smallest average error and is superior to other algorithms in recovery performance. Meanwhile, it is also shown that the running time of the DTSAMP algorithm is much lower than other algorithms, and the running time increases slowly with time. Experimental results conclusively show that sparsity estimation and step size control make DTSAMP algorithm have higher data reconstruction efficiency, and it has greater advantages in the case of large data set.

5. Conclusion

In this paper, compressed sensing theory is applied to interpolation of missing data, and a sparse adaptive reconstruction algorithm is designed. When the sparsity of missing data set is unknown, data interpolation can still be completed effectively. The algorithm is divided into two stages. In the sparsity estimation stage, the real sparsity is approximated exponentially, and then the sparsity closer to the real sparsity is obtained by dichotomy method. In the iteration stage, the real value is gradually approached by changing the step size. We also control the number of candidates introduced when selecting the support set. Therefore, both speed and precision are taken into consideration. Experiments showed that the algorithm still has a high reconstruction ability even when a large amount of data is missing, so it is reliable for the reconstruction of missing data sets.

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