ISBN: 978-981-18-1791-5
2021 the 11th International Workshop on Computer Science and Engineering (WCSE 2021)
doi: 10.18178/wcse.2021.06.042

Overview of Beamforming Research

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Abstract. Beamforming is one of the core technologies in the field of array signal processing. It is widely used in different fields, especially in wireless communication, radar, sonar, medical engineering and microphone voice array processing. For beamforming, the most important problem is the selection of optimal weighting coefficient and the comprehensive consideration of robustness. On this basis, this overview classifies and summarizes beamforming algorithms in recent years, which can be divided into five categories: based on covariance matrix reconstruction algorithm, based on steering vector correction algorithm, based on second-order cone programming algorithm, based on broadband subarray algorithm, based on neural network algorithm. Finally, the future development trend of beamforming technology is discussed.

Keywords: Beamforming, Covariance matrix, Steering vector, Second order cone programming, Wideband subarray, Neural network

1. Introduction

In recent years, the development speed of array signal processing has been significantly improved, and with its wider application, beamforming technology is becoming more and more important. Capon beamformers are proven to be theoretically optimal for situations where the covariance matrix of the received data is accurately known [1]. However, the performance of beamformers will be seriously degraded when model mismatch errors occur. At present, the improvement of the received data covariance matrix and the guidance vector to perfect the beamforming technology is still a research hotspot [2-4]. At the same time, considering the influence of many non-ideal factors in practical engineering system application, one of the hot spots in current research is the robust design of beamforming algorithm [5]. It can be seen that the core problem of beamforming technology is the optimal selection of weighting coefficients and the comprehensive consideration of robustness. In addition, with the rapid development of artificial intelligence technology, various applicable neural network models emerge one after another, and beamforming algorithm can be organically combined with artificial intelligence technology to obtain effective applications. Therefore, research on beamforming technology based on neural network will also be one of the hot spots. This paper classifies and summarizes the beamforming algorithms in recent years, and further discusses the future development trend of beamforming technology.

2. Current Status of Beamforming Technology

Beamforming is an important branch of array signal processing. In recent years, with the joint efforts of relevant researchers, various algorithms based on different array structures, based on different eigendecomposition algorithms, and applied to different small fields have been proposed, and the applicability and robustness of beamforming technology have been greatly improved. Beamforming is essentially a spatial filtering technology, the technology is the key to through certain guidelines to make the optimal weighted coefficient is obtained, and the beam direction of main lobe on the desired signal only and only receive expected direction of signal, the interference of other directions inhibition of signal and noise in maximum extent, to meet the requirements of the corresponding resolution [6-9].

Consider the n-element isometric linear array, and the array element spacing is d , and assume that the array element spacing is isotropic. The far field of a desired signal and P narrowband interference with plane wave incidence (λ is wavelength), arrival Angle θ_0 and θ_k . The snapshot data received by the array is

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represented as

$$x(t) = As(t) + n(t) \tag{1}$$

Where: $x(t)$ is the array data vector, $n(t)$ is the array noise vector, $s(t)$ is the signal complex envelope vector， $A = [a(\theta_0), a(\theta_1), \cdots, a(\theta_P)]$ is the array guidance vector， the guide vector expression of the kth signal source is $a(\theta_k) = [1, e^{j\beta_k}, \cdots, e^{j(N-1)\beta_k}](k = 1,2,\cdots,P)$, where

$$\beta_k = \frac{2\pi}{\lambda} d \sin\theta_k \tag{2}$$

The covariance matrix of the array under the background of white noise is defined as

$$R = E[x(t)x^H(t)] = AR_s A^H + \sigma_n^2 I \tag{3}$$

Where: $R_s = E[s(t)s^H(t)]$ is the signal complex envelope covariance matrix, σ_n^2 for array noise power. The beamforming output can be expressed as

$$y(t) = w^H x(t) = s(t)w^H a(\theta) \tag{4}$$

Where: w is the weighting coefficient. The basic beamforming direction diagram is shown in Figure 1.

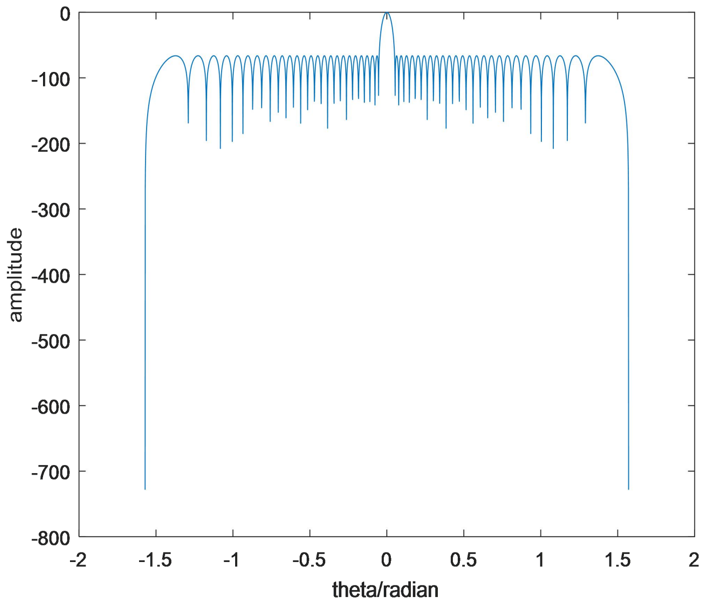

Fig. 1: Beamforming direction

Based on the above relevant beamforming principles, the following is a classification and summary of representative algorithms, which can be divided into 5 categories: covariance matrix reconstruction algorithm, guided vector correction algorithm, second-order cone programming algorithm, wide-band subarray algorithm, and neural network algorithm. Based on the general analysis of the technology development status of these five algorithms, the author summarizes his own thinking and discusses the future development trend of beamforming technology.

2.1. Covariance Matrix Reconstruction Algorithm

The core idea of the covariance matrix reconstruction algorithm is to replace the theoretical covariance matrix with the sample covariance matrix to obtain the characteristic space of the matrix, reduce the power error as much as possible, and reconstruct the covariance matrix with the integration of Capon power spectrum within a certain Angle range. Therefore, the performance of this kind of algorithm depends on the effect of covariance matrix reconstruction. If other conditions are determined, improper reconstruction of covariance matrix will have a great impact on beamforming. Huan Lu, Heng Nan, Weijie Tan[10] proposes and implements a adaptive method based on Iteration Adaptive Algorithm (IAA) the covariance matrix of refactoring robust beamforming method. This method can estimate the power spectrum accurately through

IAA and reduce the integral region to the three-dimensional ring domain.Since the algorithm does not depend on the incoherent assumption of signals, it can reduce the influence of useless information and solve the azimuth estimation and power estimation in the presence of coherent signals. At the same time, the reconstruction accuracy of the disturbance covariance matrix is significantly improved, and the SNR output is better. Wei Chen and Yun Qin[11] proposed a reduced rank beamforming algorithm based on beam domain LC-GSC. This method, linear constraint is introduced so that the beam domain covariance matrix inverse of high order power approximate equivalent to the signal subspace projection matrix, then the feature space, and the structure of the singular matrix with Generalized Sidelobe Canceller (GSC) branch in the middle of the snapshot data to complete. The algorithm can constrain the interference in a particular direction for 3 times and produce zero trapping, and the expected signal is guaranteed without distortion. Compared with the eigendecomposition, this method can effectively reduce the expected signal cancellation phenomenon, and the computational complexity is effectively reduced. The algorithm is more convenient to implement in engineering. Wei Zhang and Kun Zheng[12] proposed a full-range dimensional adaptive airspace filtering method based on two-level detection. In this method, the echo signal is segmented by distance, a temporary sampling covariance matrix is estimated, the first-level detection output is calculated, and the data samples exceeding a certain threshold are eliminated. After that, the new sampling covariance matrix and the second-level adaptive weight were calculated, and the second-level detection was carried out, and each section was processed in turn. This method can avoid the phenomenon of self-canceling signal and avoid the disadvantage that the training data does not contain interference signal. However, the effectiveness of this method has only been verified by simulation, and the actual performance needs to be verified by the measured data. Xue Zhang, Xiangpeng Zhu, Shuai Liu et al. [13] proposed a robust beamforming algorithm for quaternion matrix reconstruction.This algorithm establishes a quaternion model for the polarization sensitive array, extends the conventional reconstruction method of covariance matrix to the quaternion domain, USES the subspace method to obtain the guidance vector estimation of the disturbance signal, and then reconstructs the disturbance noise covariance matrix. The algorithm can effectively avoid the performance degradation caused by the expected signal cancellation, and the robustness of the algorithm is enhanced, and the output SNR can reach close to the optimal value. Wenbin Zhao[14] proposed a robust beamforming method based on double eigende composition. The method builds a special matrix, which is composed of signal subspace and interference directional guidance vector, and USES its internal relationship to construct a preprocessing matrix that can ensure the remaining signal power is constant and suppress the expected signal in the training data, so as to obtain the beamforming weight vector. Saeed Mohammadzadeh, Vitor H. Nascimento[15] et al. proposed a robust adaptive beamforming based on maximum entropy reconstruction of disturbance and noise covariance matrix. This algorithm USES the maximum entropy power spectrum principle to estimate all the interference power and the required signal power, so as to reconstruct the disturbance plus noise covariance matrix and the expected signal covariance matrix, with low complexity and high robustness performance.

2.2. Guidance Vector Correction Algorithm

The core of the guided vector correction algorithm is to reduce the mismatch caused by the errors between the real guide vector and the expected guide vector, and to obtain the real guide vector as much as possible by using a series of related algorithms, and to solve the problem jointly with the covariance matrix to obtain the optimal weight.

Zhiwei Yang, Pan Zhang, Ying Chen et al. [16] proposed a robust beamforming algorithm for joint iterative estimation of guidance vector and covariance matrix. In this algorithm, the initial value of the target guidance vector is obtained by sparse reconstruction, then the error optimization model is established, and then the steady state estimation value is obtained by alternating iteration operation on this basis, and then the weighted matrix is solved. The proposed algorithm can effectively improve the output SNR of the beamformers when the target guidance vector constraint deviation is encountered, but the computational complexity is high in the joint iteration process. Dajiang Ren, Chunnan Xiu, Jiawei Cheng et al. [2] proposed an improved beamforming algorithm based on the combination of guidance vector optimization and diagonal loading. The algorithm reduces the error caused by the mismatching of the guide vector by adding the interference subspace projection to the signal. The algorithm has good performance under high SNR, large fast beat and near field scattering, but a little worse under small fast beat. Bo Wang, Junwei Xie, Jing Zhang et al. [17] proposed a Capon crossover based Cross Subarray-based FDA with sinusoidlly increasing frequency offset (CSB sin-FDA) robust beamforming algorithm.The algorithm to the steering vector mismatch cases analysis of the main lobe distortion problem, the original Uniform Linear Array Frequency Diverse Array(ULA- FDA) receiving array replacement for CSB sin-FDA received array structure, based on the estimate of the error direction vector by robust Capon beamforming (RCB) algorithm amended, revised to calculate the optimal weight vector. Finally, the shape preserving of the main lobe can be realized. Peng Li, Xiang Xia, Chuanfu Yu et al. [18] proposed a robust beamforming algorithm based on weighted spatial smoothing and guidance vector estimation.In this algorithm, the submatrix is specially divided by weighted space smoothing method, and the weighted matrix is obtained by nested method to obtain the real guidance vector with uncertain range constraint. This algorithm can effectively reduce the height of the side lobe and zero trap the interference, maintain the high output SNR in the case of low fast beat number, and maintain the maximum gain of the desired signal in the case of mismatching of the guide vector. Yuhe Fang, Zhe Li , Yongxing Cao et al. [19] proposed a new robust beamforming method, which is based on the dimension expansion of guidance vector. In this method, Angle robust factors are introduced to construct extended target guidance matrix and robust generalized beam weight matrix to replace sensitive common guidance vectors. With traditional minimum variance distortionless response Minimum Variance Distortionless Response (MVDR), compared this algorithm can have lower sidelobe level, beamforming at the same time the effect will be more robust.

2.3. Second-order Cone Programming Algorithm

As a subset of convex optimization problems, second-order cone programming is often used to solve beam optimization design problems. The solution of weighted vector is transformed into the problem of maximizing the objective function under a series of linear equations and second-order cone constraints. Therefore, the key of the second order cone programming algorithm is to set different constraints to establish the optimal beam optimization model.

Qian Liu and Anyue Zhu[20] proposed a robust low-sidelobe adaptive beamforming algorithm based on second-order cone programming. The strict control of the beamside lobe stage is accomplished by optimizing the beamside lobe design and the beamside optimization problem is transformed into a second-order cone programming problem. In this algorithm, the output SNR in a large range of input SNR can still be close to the ideal situation, and the weighted vector norm can be minimized, and the robustness is the highest. However, as the side-lobe level of the beam is greatly reduced, the main lobe of the beam will be widened when the target becomes clearer, and the resolution of the imaging sonar will be reduced to some extent. Xiaoqing Wang, Haoqian Liang, Dayu Wang et al. [21] studied the conformal array beam optimization method based on convex optimization, and proposed two weighted vector norm constrained second-order cone programming methods. Method a is the lowest sidelobe main lobe smallest weighted Conformal Array Beamforming (CBF) method, the constraint rules to make the beam scanning direction of amplitude weighted value is 1, control the maximum sidelobe minimized. The second method is the high-gain weighted CBF method with the given side lobe level and the side lobe level control is added on the basis of the high gain constraint of the given side lobe level. Both methods restrict the weighted vector norm and use the internal point iteration method to suppress the high sidelobe of conformal array beamforming. Moreover, the geometry and directivity of sensor array are not required, so the sensor array is more applicable. Zhikun Chen, Kang Du, Dongliang Peng et al. [22] proposed a two-dimensional vector beamforming method based on second-order cone programming. In a two-dimensional observation plane, the method improves the receiving gain of the system and the suppression ability of the main lobe interference through the polarization matching design of the main lobe region, and superpositions the zero-notch concave surface and the orthogonal polarization constraint in the side lobe region to maximize the suppression ability of the interference. The optimal vector beam optimization problem model is established, then it is transformed into two equivalent scalar beam optimization problems and solved by second order cone programming.

2.4. Wide-band Subarray Algorithm

The core idea of the wideband subarray algorithm is to divide the original array structure into different subarray structures on the basis of the time-domain non-recursive wideband beamforming technology, and to obtain the optimal weight vector matrix by introducing the constraint conditions into the subarray beamforming. For the problem of peak value distortion of the main lobe and sidelobe gate lobe in beamforming, the following algorithm is optimized and improved from the perspective of subarray:

Jing Zhang, Junwei Xie, Bo Wang et al. [23] proposed a subarray frequency-controlled array beamforming algorithm based on bilateral small variance without distortion response. In this algorithm, one-dimensional uniform linear arrays are divided into two central symmetric subarrays with different nonlinear frequency offset, which form a spot-shaped beam at the target position, and decouple the range-angle in the directional pattern of the conventional frequency array with fixed frequency offset. The algorithm reduces the amount of computation and can effectively suppress angular dimensional inseparable interference when the number of array elements is large. However, when the whole beamforming process is virtual as two processes of transmitting and receiving beamforming, the problem of "notch" is caused by two-dimensional suppression of interference. Xiaolu Lu, Yi Zhang , Jian Yi et al. [24] combined the multi-pole array theory technology with the time-domain non-recursive wideband beamforming technology to realize the wideband beamforming of the small aperture vector hydrophone array. In this algorithm, the multipole is constructed by the difference operation of similar channel signals, and then the arbitrary beam is approximated by the Fourier series expansion of periodic function. For a certain channel, the amplitude-frequency response of the corresponding non-recursive filter at a certain frequency point needs to approximate the weighted coefficient of the narrow-band beamforming device at that frequency point. The algorithm can obtain a beam with almost no relation to frequency and a space gain of 7dB. Lizheng Zhang, Xinrong Cao, and Shiguo Li[25] proposed a gate lobe suppression method for broadband subarray beamforming. In this algorithm, the zero-point alignment technique is combined with wideband beamforming technique, and the zero-point constraint is applied to the direction of the gate lobe in the subarray beamforming in order to optimize the weight vector of beamforming. The proposed algorithm can effectively eliminate the influence of the gate lobe during the sub-array beamforming and solve the energy leakage problem so that the output SNR is basically unaffected by the phase error.

2.5. Neural Network Algorithm

The core idea of the algorithm based on neural network is to introduce the idea of deep learning into traditional beamforming, train the echo data through different neural networks, and combine the neural network technology with beamforming technology to improve the beamforming performance.

Moyu Bo, Hao Liu, Hao-chuan Chen et al. [26] proposed an adaptive beamforming algorithm based on deep neural network. In this algorithm, the deep neural network is designed by using the sectional training method. Leaky-relu activation function, Adam optimization algorithm and Dropout regularization method are applied to improve the performance of the deep neural network, and the deep neural network model is applied to pre-train the data so as to achieve fast beamforming. This algorithm makes the weight inference network of adaptive beamforming have better performance in both accuracy and generalization, and the calculation speed is about 7 ~ 8 times higher than the traditional Least Mean Sauare (LMS) algorithm.

Table 1: Comparison of beamforming algorithm performance in literature [26]

Algorithm	LMS	Without Dropout	Dropout
Mean error	0.13	0.18	0.15
Time/s	0.6	0.08	

Ziteng Wang, Xingwei Sun, Junfeng Li et al. [27] proposed and realized the minimum variance undistorted response beamforming under the approximate narrowband hypothesis. This algorithm USES bidirectional long-short memory recurrent neural network to obtain time-frequency masking, and USES relative transfer function to represent the covariance matrix of speech sound, which is used to reconstruct the covariance matrix of speech signal as rank -1 matrix. The algorithm improves the output SNR, reduces the identification error rate, and improves the robustness to noise interference and estimation error. Shuyu Zhou, Wanning Huang, Cuichun Li[28] proposed a beamforming method based on Radial Basis Function (RBF) neural network. In this method, the array weight vector obtained by the minimum mean square algorithm with a certain amount of wave arrival Angle of the same step length is taken as the training set, and then the weights of each array element in the array antenna can be quickly calculated based on the training set RBF neural network. This method can effectively reduce the computation of array weight vector and improve the real-time performance of beamforming, but there are some non-target response errors caused by neural network interpolation error. Moyu Bo, Hao Liu, Haochuan Chen et al. [29] proposed a deep neural network beamforming algorithm using knowledge distillation.This algorithm designs deep neural network based on beamforming principle, and compresses deep neural network by knowledge distillation, so that the compressed model not only has good generalization performance of the original model but also has faster computing speed. Compared with the traditional minimum mean square error algorithm, the speed of the deep neural network adaptive beamforming algorithm based on model compression is about 20 times faster in the experimental environment.

3. Development Trend of Beamforming Technology

There has always been a great deal of interest in the research of beamforming, and adaptive beamforming is the most popular research topic in various fields.According to the current research status of beamforming, there are three research directions worth further research in the future :

(1) improving and optimizing the traditional beamforming algorithm. Beamforming through beamforming is still a common method. For Capon beamformers, it is the most effective method when the received data covariance matrix is accurately known, but the model mismatch will make the covariance matrix not accurate and thus cannot be accurately estimated. Therefore, the covariance matrix is reconstructed and the guide vector is registered by means of adaptive algorithm, iterative algorithm, eigendecomposition, maximum entropy and sparse reconstruction, so as to make the expected signal distortion free as far as possible. Most of the recent related papers start from the Angle of signal, then we can try to introduce other signal processing algorithms to reconstruct the covariance matrix more accurately and effectively.

(2) The organic combination of neural network technology and beamforming technology With the gradual development and maturity of artificial intelligence technology, the advantages of neural network technology can be shown in solving various forecasting and classification problems. Therefore, the idea of deep learning can be introduced more widely, and the combination of neural network technology and beamforming technology can be applied to improve the computing speed and reduce the computational complexity. Most of the neural network models used in recent relevant papers are still basic convolutional neural networks, and the wing 5 structure and pulse neural network in image recognition have not been applied to beamforming, so the selection of neural network and the improvement of its architecture may be the research hotspot in the future.

(3) Selectivity of The Optimal weighting coefficient For beamforming technology, the key is to obtain the optimal weighting coefficient through certain criteria, so as to obtain the maximum gain of the desired direction signal. How to choose the optimal weighting coefficient is of great significance to beamforming. Most of the algorithms mentioned in recent papers only deal with the covariance matrix or guidance vector matrix after receiving the echoes in different ways, but there are few researches on the array structure itself. Considering the space-time equivalence between time-domain and frequency-domain, it can be said that the selection of the existing beamforming optimal weighting coefficients is based on time-domain non-recursive filters. There are two types of time-domain filters, in addition to non-recursive filters, recursive filters, and given that the former can be used to design the optimal weighting coefficients, perhaps the latter can also be used in beamforming. Therefore, the optimal weighting coefficient design can be considered from the aspect of array structure, and there is still room for further research.

4. Acknowledgements

This work was supported by the Space Engineering University and the School of Electronic and Optical Engineering.

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